Applied Concurrency Theory Lecture 4 : bisimulations, CCS, and pi-calculus

Hubert Garavel

Alexander Graf-Brill

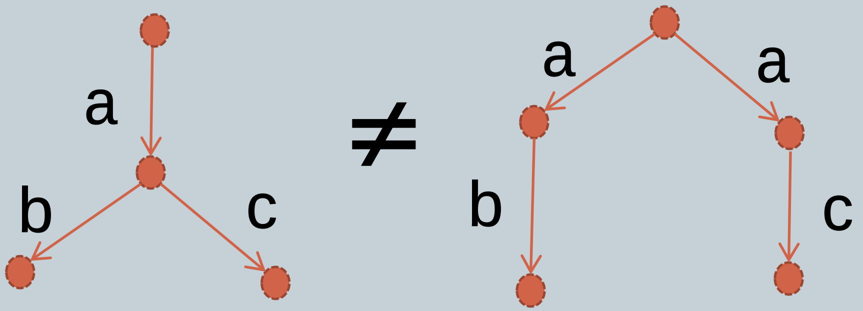
Bisimulations

Do we need equivalences at all?

- Process algebraists use equivalences because this is the only way for them to verify programs
- With operational semantics:
  - we translate (well, not to large) programs into graphs
  - we can do visual checking
  - we can do model checking
  - also, equivalences are more expensive than model checking -- roughly: O (n log n) vs O (n)
  - do we still need equivalences?
- Yes. Equivalences are useful
  - to minimize LTSs (e.g. before visual or model checking)
  - to avoid writing complex temporal logic formulas
  - to check if certain traces are accepted by an LTS
  - to fight state explosion (compositional minimization)

Why not using automata equivalence?

- Automata equivalence checks whether two automata accept the same language
  - same language = same set of accepted words (or traces)
  - this is perfect for regular expressions and compiler scanners
- This is not suitable for studying concurrency
  - comparing languages is not enough
  - two LTS may have the same language but behave differently



'coffee-vending machine' example

both LTSs recognize the same traces {a.b, a.c} but putting them in parallel with a.b generates a deadlock in the 2nd case

Do we need so many equivalences?

- In the literature, there are nearly 50 different equivalences for LTSs
- In practice, only two or three are needed:
  - strong bisimulation: preserves all properties on LTSs (well, not the number of states nor the branching factor)
  - weak bisimulation: try to eliminate or collapse sequences of τ-transitions which are not observable anyway. Branching bisimulation is a suitable weak bisimulation.
  - some divergence-preserving bisimulation
- Also useful:
  - equivalences taking time and/or probabilities into account

A critical look at CCS

Syntax of CCS

(channel, port) names: $a, b, c, \ldots$

co-names: $\bar{a}, \bar{b}, \bar{c}, \ldots$

silent action: τ

actions, prefixes: $\mu ::= a \mid \bar{a} \mid \tau$

processes:

P, Q	$::=$	0	inaction
	$\mid$	$\mu.P$	prefix
	$\mid$	$P \mid Q$	parallel
	$\mid$	$P + Q$	(external) choice
	$\mid$	$(\nu a)P$	restriction
	$\mid$	$\mathrm{rec}_K P$	process P with definition $K = P$
	$\mid$	K	(defined) process name

Dynamic semantics of CCS

- A very small number of rules

$$[\text{Act}] \quad \frac{}{\mu.P \xrightarrow{\mu} P} \qquad [\text{Res}] \quad \frac{P \xrightarrow{\mu} P' \quad \mu \neq a, \bar{a}}{(\nu a)P \xrightarrow{\mu} (\nu a)P'}$$

$$[\text{Sum1}] \quad \frac{P \xrightarrow{\mu} P'}{P+Q \xrightarrow{\mu} P'} \qquad [\text{Sum2}] \quad \frac{Q \xrightarrow{\mu} Q'}{P+Q \xrightarrow{\mu} Q'}$$

$$[\text{Par1}] \quad \frac{P \xrightarrow{\mu} P'}{P|Q \xrightarrow{\mu} P'|Q} \qquad [\text{Par2}] \quad \frac{Q \xrightarrow{\mu} Q'}{P|Q \xrightarrow{\mu} P|Q'}$$

$$[\text{Com}] \quad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \qquad [\text{Rec}] \quad \frac{P[\text{rec}_K P/K] \xrightarrow{\mu} P'}{\text{rec}_K P \xrightarrow{\mu} P'}$$

A cold look at CCS

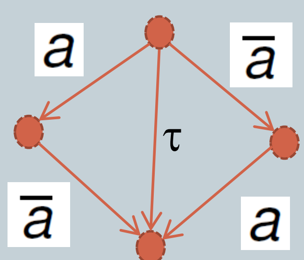
- Minimality
  - appealing in academia, but does not scale up to real problems
  - the LOTOS ISO committee added the required extensions
- Sequential composition
  - CCS action-prefix proved to be a bad language design decision
  - see Lecture 3 for a discussion (LOTOS vs LOTOS NT)
- Parallel composition
  - CCS parallel composition is worse than the one of CSP/LOTOS
  - only supports binary rendez-vous (co-names are a mistake)
  - even the binary communication is badly designed

CCS parallel composition

$$[\text{Par1}]\quad \frac{P \xrightarrow{\mu} P'}{P|Q \xrightarrow{\mu} P'|Q} \qquad\qquad [\text{Par2}]\quad \frac{Q \xrightarrow{\mu} Q'}{P|Q \xrightarrow{\mu} P|Q'}$$

$$[\text{Com}]\quad \frac{P \xrightarrow{a} P' \qquad Q \xrightarrow{\bar{a}} Q'}{P|Q \xrightarrow{\tau} P'|Q'} \qquad\qquad [\text{Res}]\quad \frac{P \xrightarrow{\mu} P' \qquad \mu \neq a, \bar{a}}{(\nu a)P \xrightarrow{\mu} (\nu a)P'}$$

- No list of gates on which to synchronize or not
- [Par1] and [Par2]: each parallel process can always evolve alone and ignore the rendez-vous!
- [Com]: the rendezvous is immediately renamed into τ impossible to observe in the LTS $\Rightarrow$ verification impossible

$a\,.\,0 \mid \bar{a}\,.0 =$ 

a restriction on a is required to force the rendezvous

CCS parallel composition: limitations

- Limitation of binary synchronization:
  how to specify (P || Q) ; R ?   (LOTOS NT semantics)

- This is a 3-party rendez-vous: P and Q wait each other to terminate and R waits to start

- CCS requires 2 additional rendezvous δ_1 and δ_2 :
  $((P \,.\, \delta_1 \mid Q \,.\, `\delta_1 \,.\, \delta_2) \setminus \delta_1 \mid `\delta_2 \,.\, R) \setminus \delta_2$
  this creates two τ-transitions in the LTS (too bad)

The pi-calculus

Motivation (1/3)

- In ‘classical’ process calculi (CCS, CSP, LOTOS…):
  - one often describes a finite set of concurrent actors
  - these actors can be (recursively) nested
  - the communication topology (i.e., gates) is fixed
  - well-adapted to hardware design, data transmission protocols

- In fact, ‘classical’ process calculi can do more:
  - dynamic creation/destruction of actors and channels
    Example: A ; hide G in (B |[G]| C) ; D
  - unbounded dynamic creation of actors
    Example: process P (N) := if N=0 then Q else (P(N-1) ||| Q)

(mixing LOTOS and LOTOS NT syntaxes)

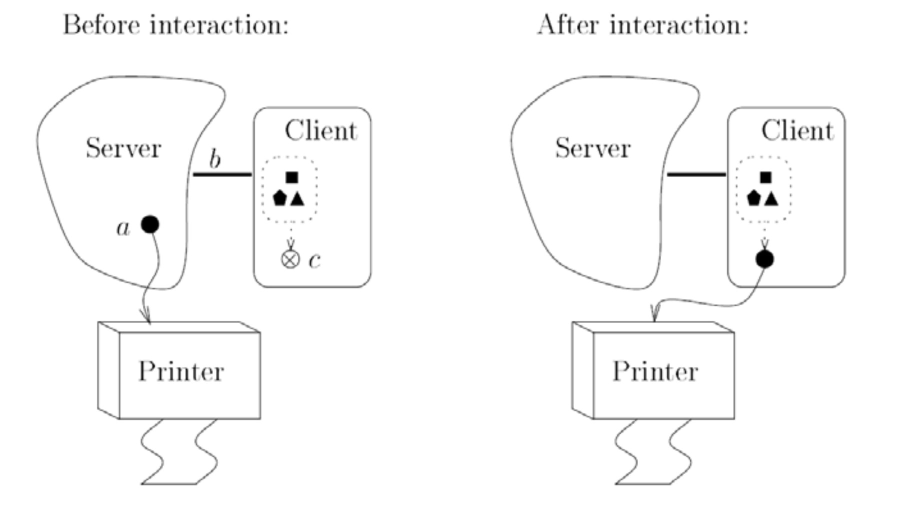
Motivation (2/3)

- ‘Mobile process calculi’ : a more radical approach
  - dynamically evolving networks
  - actors can be created/deleted dynamically
  - channels (communication links) also
  - actors can discover each other, and then communicate
  - often, they are put in relation by a third-party (‘trader’)

- Real-life examples:
  - plug-and-play devices on a network
  - mobile phones and base stations
  - object-oriented software

Motivation (3/3)

- The printer discovery example (J. Parrow):



- One approach to mobility: sending channels
  - impossible in ‘classical’ process calculi, where offers sent or received on gates only contain data values (but not gates)
  - sending processes is similar to sending channels

The pi-calculus

- Proposed by R. Milner, J. Parrow, D. Walker in the early 90s (see References)
- Defined as an extension of CCS
- Two main changes:
  - channels can be sent on channels
  - the restriction operator of CCS is technically modified
- A very influential model in academia:
  - many variants
  - some tools, such as the Mobility Workbench http://www.it.uu.se/research/group/mobility/mwb
  - some applications – basis for defining BPEL
  - see http://move.to/mobility

Syntax

Prefixes	α	$::=$	$\overline{a}x$	Output (noted a !x in LOTOS)
			$a(x)$	Input (noted a ?x in LOTOS)
			τ	Silent
Agents	P	$::=$	$\mathbf{0}$	Nil
			$\alpha\,.\,P$	Prefix
			$P + P$	Sum
			$P \mid P$	Parallel
			$\mathtt{if}\ x = y\ \mathtt{then}\ P$	Match
			$\mathtt{if}\ x \neq y\ \mathtt{then}\ P$	Mismatch
			$(\boldsymbol{\nu} x)P$	Restriction
			$A(y_1, \ldots, y_n)$	Identifier
Definitions			$A(x_1, \ldots, x_n) \stackrel{\text{def}}{=} P$	(where $i \neq j \Rightarrow x_i \neq x_j$)

Output $\overline{a}x$: also written ‘a<x>

Match and Mismatch: added later

Restriction $(\boldsymbol{\nu} x)P$: initially noted P \ a as in CCS

Static semantics

- A single 'type' of data, merging values and channels
- Variables are defined ('bound') only at 3 places:
  - x (y). P : variable y contains the data received on x
    y is visible only in P
  - (νy) P : a new channel is created and assigned to variable y
    y is visible only in P, but P may send y to other agents
    (this is called 'scope extrusion' – tricky rules)
  - A (x_1, ..., x_n) = P : parameters x_1, ..., x_n are visible in P
- *bn*(P) := bound variables defined in P : x (y) or (νy)
- *fn*(P) := all other variables used in P *(free variables)*

Dynamic semantics

$$\textsc{Tau}\ \tau.P \xrightarrow{\tau} P \qquad \textsc{Out}\ \overline{x}y.P \xrightarrow{\overline{x}y} P \qquad \textsc{In}\ x(y).P \xrightarrow{xz} P\{z/y\}$$

$$\textsc{Sum}\ \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \xrightarrow{\alpha} P_1'} \qquad \textsc{Par}\ \frac{P_1 \xrightarrow{\alpha} P_1'}{P_1|P_2 \xrightarrow{\alpha} P_1'|P_2}\ \text{if } bn(\alpha) \cap fn(P_2) = \emptyset$$

$$\textsc{Com}\ \frac{P_1 \xrightarrow{\overline{x}y} P_1' \quad P_2 \xrightarrow{xy} P_2'}{P_1|P_2 \xrightarrow{\tau} P_1'|P_2'} \qquad \textsc{Close}\ \frac{P_1 \xrightarrow{\overline{x}(y)} P_1' \quad P_2 \xrightarrow{xy} P_2'}{P_1|P_2 \xrightarrow{\tau} (\nu y)(P_1'|P_2')}\ \text{if } y \notin fn(P_2)$$

$$\textsc{Res}\ \frac{P \xrightarrow{\alpha} P'}{(\nu x)P \xrightarrow{\alpha} (\nu x)P'}\ \text{if } x \notin n(\alpha) \qquad \textsc{Open}\ \frac{P \xrightarrow{\overline{x}y} P'}{(\nu y)P \xrightarrow{\overline{x}(z)} P'\{z/y\}}\ \text{if } x \neq y, z \notin fn((\nu y)P')$$

$$\textsc{Match}\ \frac{P \xrightarrow{\alpha} P'}{[x = x]P \xrightarrow{\alpha} P'} \qquad \textsc{Mismatch}\ \frac{P \xrightarrow{\alpha} P'}{[x \neq y]P \xrightarrow{\alpha} P'}\ \text{if } x \neq y$$

$$\textsc{Ide}\ \frac{P\{y_1/x_1, \ldots, y_{r(A)}/x_{r(A)}\} \xrightarrow{\alpha} P'}{A(y_1, \ldots, y_{r(A)}) \xrightarrow{\alpha} P'}\ \text{if } A(x_1, \ldots, x_{r(A)}) \stackrel{\text{def}}{=} P$$

Example

Taken from Mateescu-Salaün IFM 2010 paper (see references)

$$
\begin{aligned}
Main &= (\nu\ req, a, b, c)(Client(req, a, b, c) \mid Dispatcher(req) \mid \\
&\quad Server(a) \mid Server(b) \mid Server(c)) \\
Client(req, a, b, c) &= (\nu x)(\overline{request}\ a.\overline{req}\langle a, x\rangle.ClientAux(req, a, a, b, c, x)) + \\
&\quad (\nu x)(\overline{request}\ b.\overline{req}\langle b, x\rangle.ClientAux(req, b, a, b, c, x)) + \\
&\quad (\nu x)(\overline{request}\ c.\overline{req}\langle c, x\rangle.ClientAux(req, c, a, b, c, x)) \\
ClientAux(req, k, a, b, c, x) &= x(info).(\overline{x}\ purchase.\overline{purchase}\ k.0 + \\
&\quad \overline{x}\ refuse.\overline{refuse}\ k.Client(req, a, b, c)) \\
Dispatcher(req) &= req(k, x).\overline{k}\ x.Dispatcher(req) \\
Server(k) &= k(x).\overline{x}\ info.x(decision).Server(k)
\end{aligned}
$$

The PIC2LNT tool

PIC2LNT (1/2)

- A recent translator developed at INRIA Grenoble

- Input language: PIC
  - pi-calculus
  - with a machine-readable syntax (from Mobility Workbench)
  - extended with data values (= ‘applied pi-calculus’)

- Output: LOTOS NT program

- A script named ‘pic2bcg’ automates the translation
  PIC $\rightarrow$ LOTOS NT $\rightarrow$ LOTOS $\rightarrow$ Petri nets $\rightarrow$ LTS

PIC2LNT (2/2)

- The PIC language
  - defined in the PIC2LNT manual page (see References)
  - the data types and value expressions are those of LOTOS NT

- The translation approach:
  - most pi-calculus tools do symbolic proofs on the terms
  - pic2lnt works by state space exploration (= explicit-state enumeration = reachability analysis)
  - limitation: only works for finite-state models
  - $\Rightarrow$ bounding channels, data types, '!' operator
  - BUT enables to study non-trivial mobile programs

A few notes

- Caution: ‘t’ means τ (contrary to ‘i’ in LOTOS/NT)
- The restriction operator ν must be written ‘**new**’
- Emissions $\bar{x}$ have to be noted ’x
- Emitted parameters must be bracked with < and > even when there is only a single parameter
- Received parameters must be bracked with (and) even when there is only a single parameter
- There are no channel declarations: beware of typos
  - exploit: at any place, you can easily insert a ’debug event

More notes

- In the LTS obtained, the labels carry extra offers
  - for instance: !FALSE or !TRUE
  - this is an artefact of the translation to LOTOS NT
  - (perhaps the pic2bcg script could remove them)
- The translation implements the creation of new channels by giving unique numbers
  - example: (new y) 'x<y> may generate a transition: X !Y(41)
  - don't worry if the counter is not increasing one by one
- Restriction hides the synchronizations ☹
  - one cannot observe them in the LTS (only τ-transitions can be seen)
  - add extra events if needed

Today's challenge

Your first pi-calculus program (1/2)

- Find the paper about PIC2LNT published at IFM 2010 (see References below)
- Copy-and-paste in a file named ‘disp.pic’ the pi-calculus example given page 11
- Convert it to machine-readable notations:
  - replace each ν symbol by the **new** keyword
  - replace emissions $\bar{x}$ with ’x
  - restore the < and > symbols around emissions of multiple channels; add them for emissions of single channels
  - same with (and) for receptions
  - finally, replace the 0 with nil (0 is not documented in the manual page, yet seems to be accepted)

Your first pi-calculus program (2/2)

- Perform the translation PIC → LOTOS NT → LOTOS → Petri nets → LTS by typing:
  - $ pic2bcg disp.pic
  - if it does not compile properly, fix the mistakes
- Visualize the file ‘disp.bcg’ obtained
  - $ bcg_edit disp.bcg
- Compare it to the picture given page 11
- Minimize it using strong bisimulation to remove ‘duplicated’ parts of the LTS
  - $ bcg_min disp.bcg
  - $ bcg_edit disp.bcg
- Send your file ‘disp.pic’ and the PostScript file to Alexander (possibly with comments if you observe a difference with the picture of the paper)

References

Pi-calculus bibliography

- J. Parrow. *An introduction to the pi-calculus*. Chapter of the Handbook of Process Algebra, 2001. http://user.it.uu.se/~joachim/intro.ps Especially sections 1, 2.1, (2.1), 2.3, 4, and 6.
- U. Nestmann. *Welcome to the Jungle: A subjective guide to mobile process calculi*, 200x. http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.89.6712

Pi-calculus bibliography

- R. Milner, J. Parrow, D. Walker. *A calculus of mobile processes (parts I and II)*. Information and Computation, vol. 100, num. 1, 1992.

- R. Milner. *Elements of interaction: Turing award lecture*. http://dl.acm.org/citation.cfm?id=151240

- On-line resources: http://move.to/mobility

Tools for the pi-calculus

PIC2LNT translator, by R. Mateescu and G. Salaün, 2010-12.

In your VM, directory $HOME/Desktop/PIC2LNT

- Reference documentation:
  The PIC2LNT manual page
  in your VM, directory $HOME/Desktop/PIC2LNT/man/pdf

- If you want details on the translation:
  R. Mateescu and G. Salaün. *Translating Pi-Calculus into LOTOS NT*. IFM 2010
  in your VM, directory $HOME/Desktop/PIC2LNT/doc/pdf
  (caution: their version of LOTOS NT is highly simplified)