Nested-Units Petri Nets

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Outline

- Introduction
- The NUPN model
- The unit-safeness property
- Some expressiveness results
- The place-fusion abstraction
- Optimized encoding of markings
- Software support for NUPNs
- Conclusion

Three controversial equations in concurrency theory

Controversial equation #1

from: **R. van Glabbeek** and **F. Vaandrager**.
Petri Net Models for Algebraic Theories of Concurrency (PARLE, 1987)

(for all a, b, c : actions) a.(b +c) = a.b + a.c ?

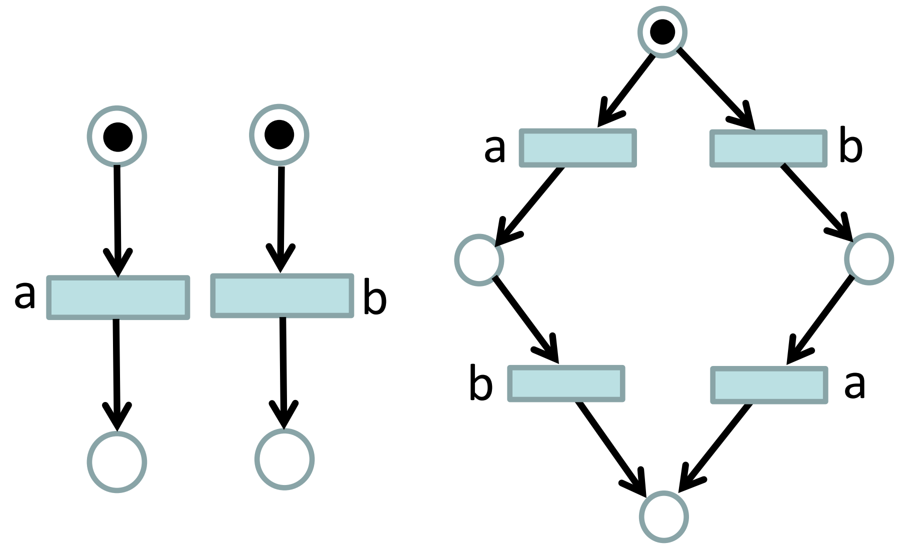
- If the answer is yes
  - linear-time semantics
- If the answer is no
  - branching-time semantics

Controversial equation #2

from: **R. van Glabbeek** and **F. Vaandrager**.
Petri Net Models for Algebraic Theories of Concurrency (PARLE, 1987)

(forall a, b : actions) a || b = a.b + b.a ?

- If the answer is yes
  - interleaving semantics
- If the answer is no
  - true concurrency
  - Petri nets can distinguish
  - (Mazurkiewicz traces and Winskel event structures can too)



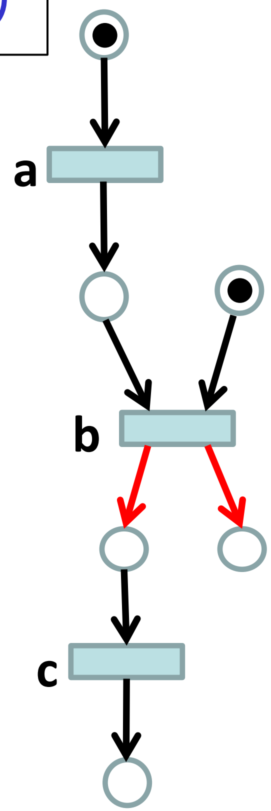
A 3rd controversial equation…

(forall a, b, c) $(a.b) \,||_b\, (b.c) = (a.b.c) \,||_b\, b$?

see also: **G. Boudol, I. Castellani, M. Hennessy, A. Kiehn**

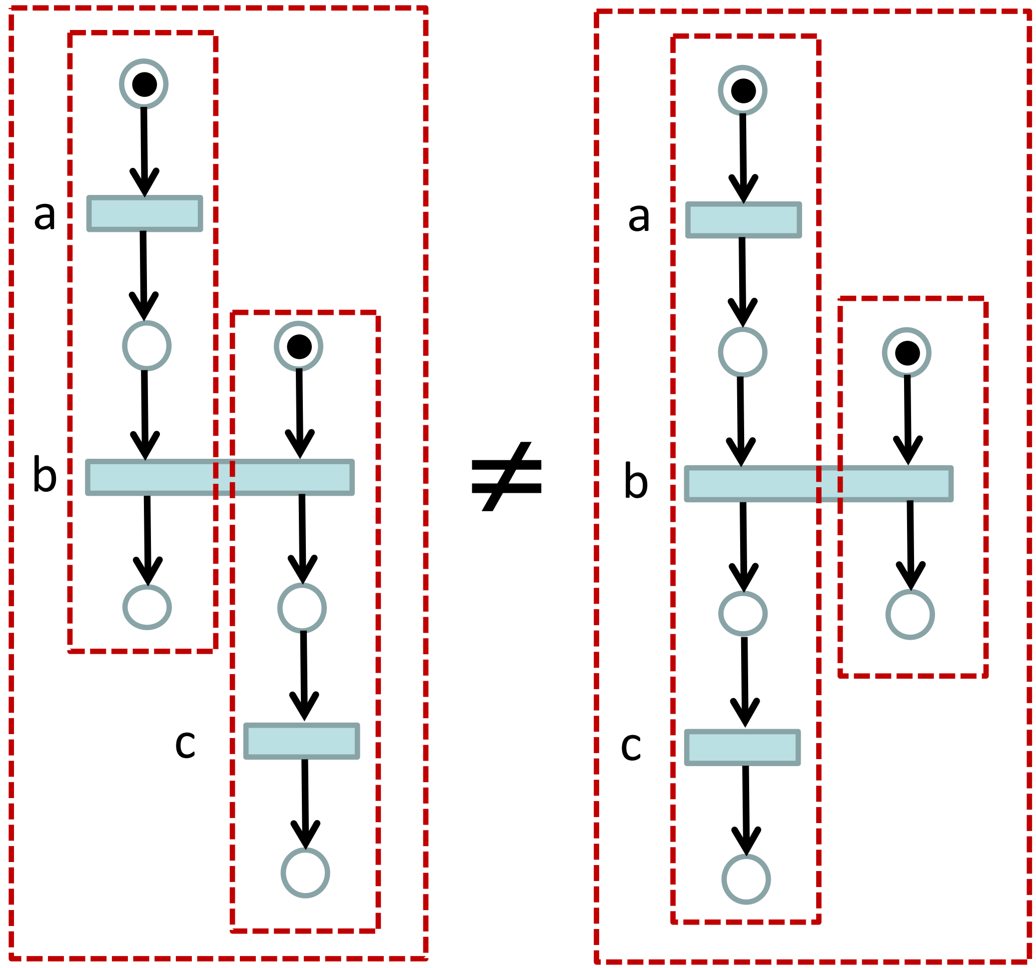
A theory of processes with localities (Form. Asp. Comp. 1994)

- Interleaving semantics:
  - they are the same (i.e., a.b.c)
- Petri nets:
  - they are also the same -------------------------->
  - no way to indicate that a and c are not on the same side
  - Petri nets preserve **concurrency**, not **locality**



How to model locality and hierarchy?

- Places that belong to the same sequential process are enclosed into "units"
- Units can be recursively nested at an arbitrary depth



$(a.b)\ ||_b\ (b.c)$ $\quad$ $(a.b.c)\ ||_b\ b$

The NUPN model
(NUPN = Nested-Unit Petri Nets)

NUPN definition

- Extension of elementary nets (all arc weights = 1)
- NUPN = 8-tuple (P, T, F, M_0, U, u_0, $\sqsubseteq$, unit)
  - Elements 1-4 of this tuple are standard

Definition 1. *A (marked) Nested-Unit Petri Net (acronym: NUPN) is a 8-tuple* $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *where:*

1. *P is a finite, non-empty set; the elements of P are called* places.
2. *T is a finite set such that* $P \cap T = \varnothing$*; the elements of T are called* transitions.
3. *F is a subset of* $(P \times T) \cup (T \times P)$*; the elements of F are called* arcs.
4. M_0 *is a subset of P;* M_0 *is called the* initial marking.

NUPN definition

- NUPN = 8-tuple (P, T, F, M_0, U, u_0, $\sqsubseteq$, unit)
  - Elements 5-8 of these tuples are novel:
    (5,6,7): tree of units + (8): mapping: place $\rightarrow$ unit

5. U *is a finite, non-empty set such that* $U \cap T = U \cap P = \varnothing$*; the elements of* U *are called* units.
6. u_0 *is an element of* U*;* u_0 *is called the* root unit.
7. $\sqsubseteq$ *is a binary relation over* U *such that* $(U, \sqsupseteq)$ *is a tree with a single root* u_0*, where* $(\forall u_1, u_2 \in U)\ u_1 \sqsupseteq u_2 \stackrel{\text{def}}{=} u_2 \sqsubseteq u_1$*; thus,* $\sqsubseteq$ *is reflexive, antisymmetric, transitive, and* u_0 *is the greatest element of* U *for this relation; intuitively,* $u_1 \sqsubseteq u_2$ *espresses that unit* u_1 *is transitively nested in or equal to unit* u_2.
8. unit *is a function* $P \rightarrow U$ *such that* $(\forall u \in U \setminus \{u_0\})\ (\exists p \in P)\ \mathsf{unit}(p) = u$*; intuitively,* $\mathsf{unit}(p) = u$ *expresses that unit* u *directly contains place* p.

Analogy with known data structures

- File systems
  - unit → directory
  - place → file

  Directories can be recursively nested at arbitrary depth

  Each directory may (or not) contain files

- XML documents
  - unit → element
  - place → attribute

  (contrary to XML, the order of elements is not significant)

Units are not boxes...

- A NUPN units encapsulates places only

  This is different from "boxes" (or "subnets") that encapsulate places, transitions, and arcs

- Another key difference is parallel composition:
  - 2 boxes in parallel $\rightarrow$ 1 box
  - 2 units in parallel $\rightarrow$ 3 units

Execution rules ("token game")

- The usual firing rules of Petri nets are unchanged
- Units are totally orthogonal to transitions
- Yet, units allow markings to be structured:

$$\text{places}\,(u) \stackrel{\text{def}}{=} \{p \in P \mid \text{unit}\,(p) = u\} \qquad \widetilde{U} \stackrel{\text{def}}{=} \{u \in U \mid \text{places}\,(u) \neq \varnothing\}$$

Proposition 1. *Let* $(P, T, F, M_0, U, u_0, \sqsubseteq, \text{unit})$ *be a NUPN. The family of sets* $\text{places}\,(u)$, *where* $u \in \widetilde{U}$, *is a partition of* P.

$$M \triangleright u \stackrel{\text{def}}{=} M \cap \text{places}\,(u)$$

Proposition 2. *Let* $(P, T, F, M_0, U, u_0, \sqsubseteq, \text{unit})$ *be a NUPN. Any marking* M *can be expressed as* $M = (M \triangleright u_1) \uplus \ldots \uplus (M \triangleright u_n)$, *where* $u_1, \ldots, u_n$ *are the units of* $\widetilde{U}$, *and where* $\uplus$ *denotes the disjoint set union.*

The unit-safeness property

Unit-safeness property

Disjonction of two units

$\mathsf{disjoint}(u_1, u_2) \stackrel{\text{def}}{=} (u_1 \not\sqsubseteq u_2) \wedge (u_2 \not\sqsubseteq u_1)$ *characterizes pairs of units neither equal nor nested one in the other.*

Unit safeness of a marking

Definition 5. *Let* $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *be a NUPN. A marking* $M \subseteq P$ *is said to be* unit safe *iff it satisfies the predicate defined as follows:* $\mathsf{unit\text{-}safe}(M) \stackrel{\text{def}}{=} (\forall p_1, p_2 \in M)\ (p_1 \neq p_2) \Rightarrow \mathsf{disjoint}(\mathsf{unit}(p_1), \mathsf{unit}(p_2))$*; that is, all places of a unit-safe marking are contained in disjoint units.*

Unit safeness of a NUPN

Definition 6. *Let* $N = (P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *be a NUPN.* N *is said to be* unit safe *iff it is safe and all its reachable markings are unit safe.*

Note: Using P/T nets rather than elementary nets, the safeness condition (i.e., contact freeness) would not be needed to ensure that strict-firing and weak-firing rules coincide

Unit safeness ⇒ local mutual exclusion

Proposition 3. *Let* $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *be a NUPN. For each marking* M *and unit* u, $\mathsf{unit\text{-}safe}(M) \Rightarrow card(M \triangleright u) \leq 1$*; that is, a unit-safe marking cannot contain two different local places of the same unit.*

- In each unit, local places are mutually exclusive
- In terms of linear algebra:

$$\sum_{p \in \mathsf{places}\,(u)} x_p \leq 1$$

- So, unit safeness implies safeness (in fact, from the definition)
- These are not S-invariants, but inequalities
  - *because a given unit may lose its token*

Unit safeness ⇒ hierarchical mutual exclusion

Proposition 4. *Let* $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *be a NUPN. For each marking* M *and units* (u, u')*, one has:* $\mathsf{unit\text{-}safe}(M) \wedge (M \triangleright u \neq \varnothing) \wedge (u' \sqsubset u \vee u \sqsubset u') \Rightarrow (M \triangleright u' = \varnothing)$*; that is, if a unit-safe marking contains a local place of some unit* u*, it contains no local place of any ancestor or descendent unit* u' *of* u*.*

- Parent and children units are mutually exclusive
  - If a parent has a token, children have no token
  - If a child has a token, parents have no token

$$\boxed{B_1 \;;\; (\; \boxed{B_2} \;||\; \boxed{B_3} \;)\;;\; B_4}$$

Linear-algebraic characterization

- Unit-safeness ⇔ system of linear inequalities

Proposition 6. *Let* $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *be a safe NUPN. N is unit safe iff any reachable marking M satisfies the following system of inequalities:*

$$(\forall u \in \widetilde{U})\ (\forall u' \in \widetilde{U} \mid u \sqsubseteq u')\ \sum_{p \in \mathsf{places}\,(u) \cup \mathsf{places}\,(u')} x_p \leq 1 \qquad (I_{u,u'})$$

where each variable x_p *is equal to 1 if place p belongs to M, or 0 otherwise.*

- Again, these are inequalities, not S-invariants

Some expressiveness results

How restrictive is unit safeness?

- Unit safeness is an (optional) property of NUPNs
- Unit-safe NUPNs are well-adapted to encode:
  - (nested) **co-begin/co-end** programming schemes
  - **process calculi** terms (without recursion through parallel composition)
- Unit-safe NUPNs can also express:
  - all safe elementary nets
  - all nets having a state-machine decomposition

This is shown by translation to unit-safe NUPNs

Elementary safe net → unit-safe NUPN

Proposition 8. *Let* (P, T, F, M_0) *be any ordinary, safe P/T net (i.e., a safe elementary net). There exists at least one 4-tuple* $(U, u_0, \sqsubseteq, \mathsf{unit})$ *such that* $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ *is a unit-safe NUPN.*

- NUPNs generalize safe elementary nets
- *N* places → *N+1* units
  - *N* units, one single place in each unit
  - one root unit having no local place

State-machine net → unit-safe NUPN

Proposition 9. *Let (P, T, F, M_0) be any ordinary P/T net possessing a state-machine decomposition. There exists at least one 4-tuple $(U, u_0, \sqsubseteq, \mathsf{unit})$ such that $(P, T, F, M_0, U, u_0, \sqsubseteq, \mathsf{unit})$ is a unit-safe NUPN.*

- NUPNs generalize state machines
- *N* state machines → *N+1* units
  - *N* units, one per state machine
  - one root unit having no local place

The place-fusion abstraction

Place-fusion abstraction

- Idea:
  - merge all places of each unit into a single place
  - perform reachability exploration on this abstracted net
- Advantages:
  - complexity reduction when units have many places
  - useful to determine concurrent units [Garavel-Serwe-06]
- Place-fusion abstraction:
  - preserves the NUPN property
  - but does not preserve safeness, nor unit safeness

Optimized encodings for markings

Gains due to safeness /unit safeness

- For safe nets: markings can be encoded with one bit per place (rather than one integer per place)
- For unit-safe nets: further reductions are possible
  - local reductions (in each unit)
    N places in a unit $\Rightarrow$ N+1 local states
    $\lceil \log 2\ (N+1) \rceil$ or $\lceil \log 2\ (N) + 1 \rceil$ bits
  - hierarchical reductions (between parent/children units)
    "vertical" overlapping between:
    — the bits encoding the N places of a unit
    — the bits encoding all sub-units of this unit

Statistical results

- 5 encoding schemes compared on > 3500 NUPNs
- Best encoding: local + hierarchical reductions applied recursively on the tree of nested units
- Number of bits reduced by more than 60%

scheme	*overlapping*	*number of bits or Boolean variables*	*average size*
(a)	no	$\sum_{i\in\{1,\ldots,n\}} N_i$ (i.e., N)	100.00%
(b)	no	$\sum_{i\in\{1,\ldots,n\}} \lceil \log_2(N_i + 1) \rceil$	40.52%
(c)	no	$\sum_{i\in\{1,\ldots,n\}} (\lceil \log_2(N_i) \rceil + 1)$	46.44%
(b)	yes	$\nu(u_0)$ with $\mathsf{leaf}(u_j) \Rightarrow \nu(u_j) = \lceil \log_2(N_j + 1) \rceil$	39.35%
(c)	yes	$\nu(u_0)$ with $\mathsf{leaf}(u_j) \Rightarrow \nu(u_j) = \lceil \log_2(N_j) \rceil + 1$	44.94%

H-W-B codes

- A useful metrics to measure NUPN complexity
- Metrics: a triple of integers, noted H-W-B
  - H is the height of the tree of nested units
    (the root unit does not count if it has no local place)
  - W is the width of the tree of nested units, i.e., the number of leaf units
    (if the NUPN is unit safe, W gives an upper bound on the number of tokens present in reachable markings)
  - B is the number of bits needed to represent markings using the best recursive encoding
- If B = number of places, the code is noted --B (H=1, W=B)

Software support for NUPNs

The ".nupn" file format

- Textual format used by CADP tools
- Concise, human-readable, easy to read and parse

`!creator caesar`	*The NUPN was created by the CÆSAR tool.*
`!unit_safe`	*The creator tool warrants that unit-safeness holds.*
`places #5 0...4`	*There are 5 places numbered from 0 to 4.*
`initial place 0`	*The initial marking contains only place 0.*
`units #3 0...2`	*There are 3 units numbered from 0 to 2.*
`root unit 0`	*The root unit is unit 0.*
`U0 #1 0...0 #2 1 2`	*Unit 0 contains 1 place (0) and 2 sub-units (1, 2).*
`U1 #2 1...2 #0`	*Unit 1 contains 2 places (1, 2) and no sub-unit.*
`U2 #2 3...4 #0`	*Unit 2 contains 2 places (3, 4) and no sub-unit.*
`transitions #3 0...2`	*There are 3 transitions numbered from 0 to 2.*
`T0 #1 0 #2 1 3`	*Trans. 0 has 1 input place (0) and 2 output places (1, 3).*
`T1 #1 1 #1 2`	*Trans. 1 has 1 input place (1) and 1 output place (2).*
`T2 #1 3 #1 4`	*Trans. 2 has 1 input place (3) and 1 output place (4).*

The NUPN extension for PNML

- PNML: ISO standard for Petri nets (2011)
- A NUPN-specific extension of PNML has been defined for the Model Checking Contest

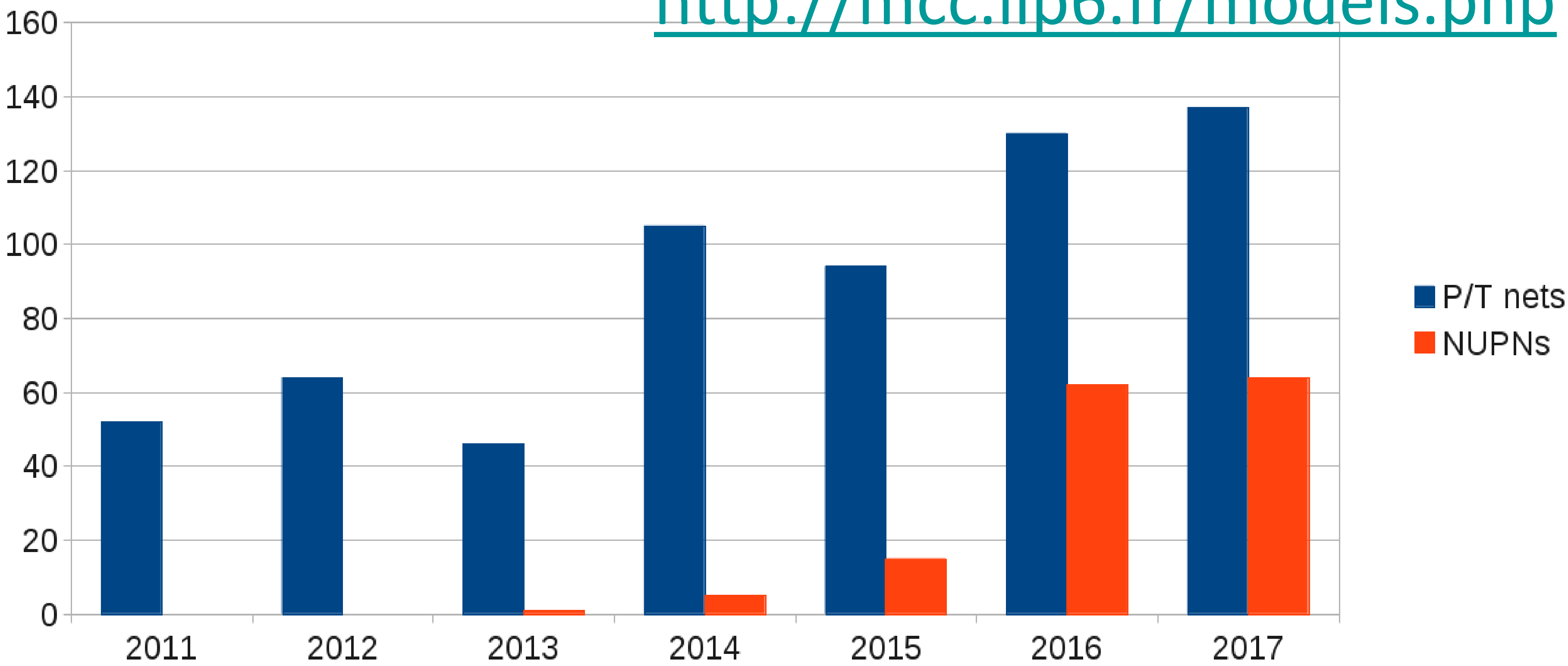
```
<toolspecific tool="nupn" version="1.1">
   <size places="5" transitions="3" arcs="7"/>
   <structure units="3" root="u0" safe="true">
      <unit id="u0" >
         <places>p0</places>
         <subunits>u1 u2</subunits>
      </unit>
      <unit id="u1">
         <places>p1 p2</places>
         <subunits/>
      </unit>
      <unit id="u2">
         <places>p3 p4</places>
         <subunits/>
      </unit>
   </structure>
</toolspecific>
```

http://mcc.lip6.fr/nupn.php

Where to find NUPN examples?

- MCC (Model Checking Contest)

http://mcc.lip6.fr/models.php



In total: 147 NUPNs out of 628 P/T nets (23%)

Where to find NUPN examples?

- VLPN (Very Large Petri Nets) *(in preparation)*

  http://cadp.inria.fr/resources/vlpn

  350 realistic benchmarks collected from diverse sources: CHP, EXP, Fiacre, LOTOS, LNT, applied pi-calculus, etc.

  - Group 1: nets containing redundant units
  - Group 2: nets containing disconnected places or transitions
  - Group 3: unsafe nets
  - Group 4: nets having one single unit — code: 1-1-B
  - Group 5: unstructured nets — code: - - B
  - Group 6: communicating automata — code: 1-W-B, with W≥2
  - Group 7: pseudo-communicating automata — code: 2-W-B
  - Group 8: genuine NUPNs (concurrency + hierarchy) — code: H-W-B , with W≥3

How to produce NUPNs?

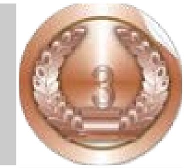
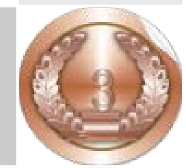
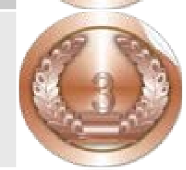
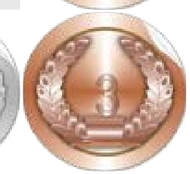
- From "flat" Petri nets:
  - PNML2NUPN (Lom Messan Hillah, Paris)
  - translation PNML → NUPN (applies Prop. 8)
- From networks of communicating automata:
  - EXP.OPEN (Frédéric Lang, Grenoble)
  - translation EXP networks → NUPN (applies Prop. 9)
- From process calculi:
  - CAESAR (Hubert Garavel, Grenoble)
  - translation LOTOS → NUPN (more involved!)

How to analyze NUPNs?

- CAESAR.BDD (Hubert Garavel, Grenoble)
  - syntax /static semantics checks on ".nupn" files
  - structural and behavioural properties using BDDs
  - translation NUPN $\rightarrow$ PNML
- CAESAR.SDD (Alexandre Hamez, Toulouse)
- GreatSPN (Elvio Amparore, Torino)
- ITS-TOOLS (Yann Thierry-Mieg, Paris)
- LoLA (Karsten Wolf & Torsten Liebke, Rostock)
- LTSmin (Jeroen Meijer & Jaco van de Pol, Twente)
- PNMC (Alexandre Hamez, Toulouse)

Model Checking Contest 2017

- 10 competing tools
- 4 tools supporting NUPNs
  - they won all golden medals
  - they won 73% of medals
- Details: http://mcc.lip6.fr

MCC category	1st best tool	2nd best tool	3rd best tool
State Space		X	
Upper Bound			X
Reachability		X	
CTL Formulas		X	
LTL Formulas			

Conclusion

Benefits of NUPNs

- They store more information than other models:
  - LTS: no concurrency – no locality – no hierarchy
  - Petri nets: concurrency – no locality – no hierarchy
  - NUPN: concurrency + locality + hierarchy
- NUPNs are easy to produce from process calculi, high-level nets, communicating automata, etc.
- NUPNs allow significant savings in state-space generation (60% less bits/Boolean variables)
- NUPNs smoothly integrate with existing tools: no major software rewrite needed

Challenging open issues

- Dedicated algorithms exploiting NUPN structure
  - to efficiently decide if a NUPN is unit-safe
  - to compute behavioural properties: deadlocks, etc.
  - to enhance partial-order / stubborn-set reductions
- Conversion of "flat" Petri nets to "optimal" NUPNs
  - "hierarchical" decomposition into state machines
  - goal: less units, more places per unit, maximal nesting
- NUPNs extended to support multiple tokens
  - relax unit-safeness constraint $\Rightarrow$ new flow relations
  - useful to encode process calculi with parallel recursion