
Partial order reductions using compositional confluence detection

Frédéric Lang
Radu Mateescu

INRIA Rhône-Alpes / VASY

<http://www.inrialpes.fr/vasy>



Context (1/2)

- **Explicit state verification of concurrent systems**
 - Parallel composition of asynchronous processes
 - Synchronisation or interleaving of communication actions
 - Systematic exploration of the behaviour graph
- **Several techniques to palliate state explosion**
 - *Compositional verification* : apply property preserving reductions to the graphs of the composed processes
 - *Partial order reductions* : avoid interleavings that are useless with respect to the properties under verification
 - *On-the-fly verification* : only explore states when necessary to evaluate the property under verification



Context (2/2)

- Those techniques can be combined
 - CADP toolbox (<http://www.inrialpes.fr/vasy/cadp>)
 - Open/Caesar environment
 - Exp.Open tool
- This talk presents two variants of a *new partial order reduction technique*, one preserving deadlocks and one preserving branching equivalence, based on a *compositional analysis* of the composed processes



Partial order reductions

persistent sets family [Godefroid, Valmari, Peled]

- Roots in communicating automata theory
- Operations are *dependent* if there can be some state in which they do not commute
- Find a subset S of the *operations* enabled in the current state such that every operation $\notin S$ and *dependent* on an operation $\in S$ cannot be enabled before an operation $\in S$ is fired
- *Deadlocks* are preserved if operations $\notin S$ are postponed
- *Visible traces* or *branching equivalence* can be preserved under additional conditions



Partial order reductions

τ -confluence family [Groote, van de Pol, Ying]

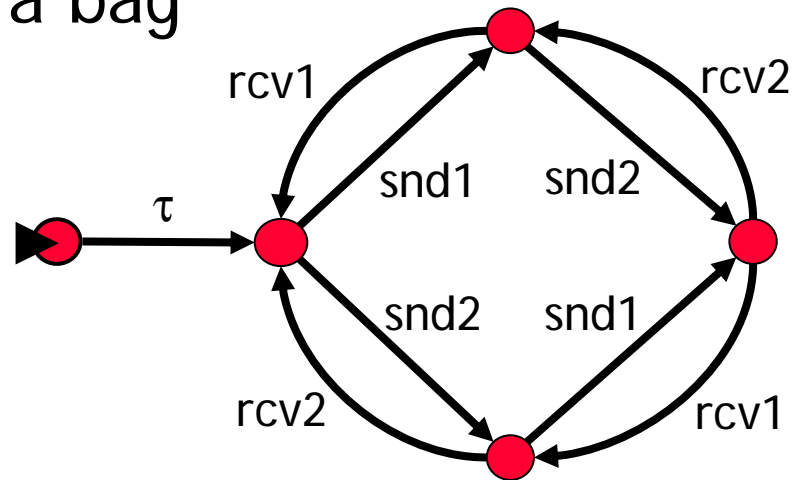
- Roots in process algebra theory
- Find invisible (τ) transitions commuting with all other transitions
- *Branching equivalence* is preserved if transitions in choice with τ -confluent transitions are postponed
- Symbolic and/or (on-the-fly) explicit state detection tools exist

This talk combines persistent sets and τ -confluence



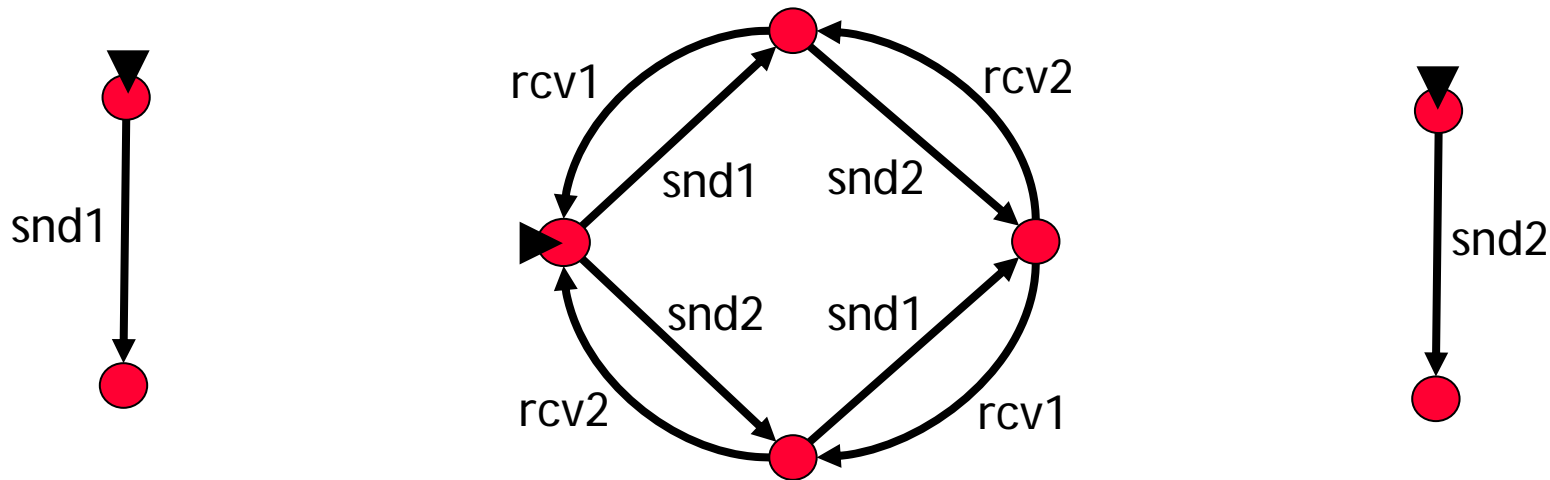
The network model (1/3)

- The model we use to represent concurrent systems
- Each process is described by a graph
- Each transition is labeled by a *visible communication action* or an *invisible action* τ
- **Example:** a bag



The network model (2/3)

- Graphs are composed using *synchronization rules*
- **Example**: Network N

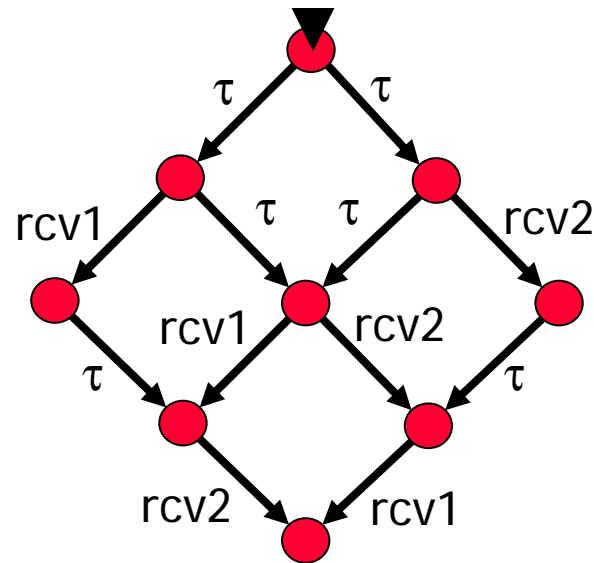


Rules: $(\bullet, \text{rcv1}, \bullet) \rightarrow \text{rcv1}$ $(\text{snd1}, \text{snd1}, \bullet) \rightarrow \tau$
 $(\bullet, \text{rcv2}, \bullet) \rightarrow \text{rcv2}$ $(\bullet, \text{snd2}, \text{snd2}) \rightarrow \tau$



The network model (3/3)

- Network semantics = product of composed graphs
- **Example**: semantics of N (previous slide)



- Reasonable restrictions on τ actions guarantee that branching equivalence is a congruence for networks (no synchronisation, no cut, and no renaming of τ actions)



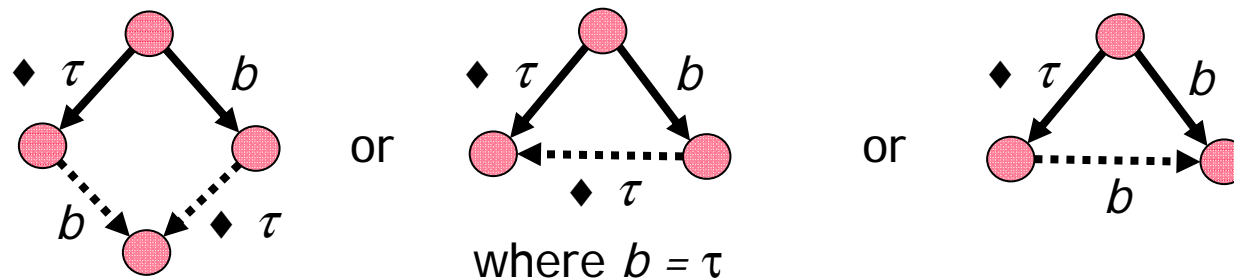
Persistent sets for networks

- Two operations are *dependent* if there is some state in which they may not commute
 - For networks, *operation* = *synchronization rule*
 - Two rules $(a_1, \dots, a_n) \rightarrow a$ and $(b_1, \dots, b_n) \rightarrow b$ are *dependent* if $(\exists i \in 1..n) a_i \neq \bullet \wedge b_i \neq \bullet$
 - Indeed, *in this case and only in this case*, there can be a state where one rule disables the other
- Persistent set construction for networks is described in [Lang-05]



τ -confluence

- Definition of *partial strong τ -confluence* by Grooten & van de Pol (τ -confluence for short in this talk)
- A transition is *τ -confluent* ($\overset{\blacklozenge}{\longrightarrow}^{\tau}$) if:



- τ -confluent transitions can be *prioritized* as long as they do not close a circuit
- This preserves branching equivalence



τ -confluence for networks

- τ -confluence can be **eliminated in composed graphs**
 - **Correct** because τ -confluence elimination preserves branching equivalence
 - But **useless** if graphs are minimized for branching
- τ -confluence can be **eliminated on-the-fly while computing the product graph**
 - Efficient tools exist (**EXP.OPEN/REDUCTOR** tools of **CADP**)
 - But **cost increases non-linearly** with the size of the product graph



Compositional confluence detection

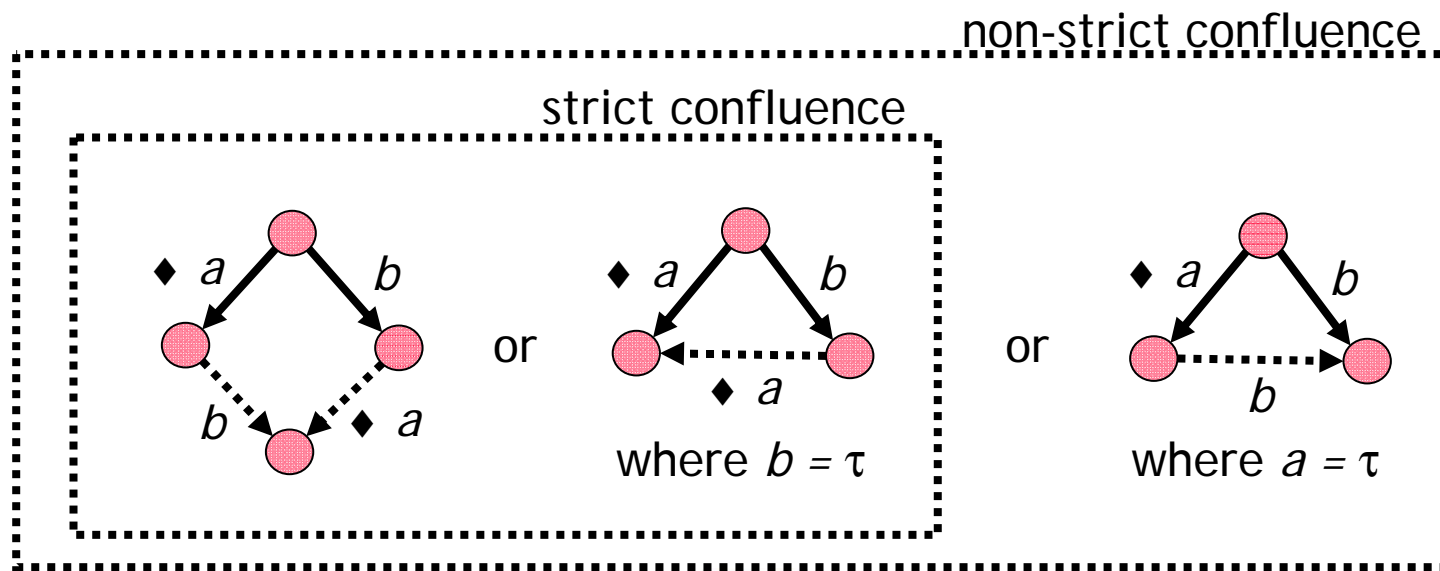
We present *Compositional Confluence Detection* (CCD)

- CCD removes some τ -confluent transitions that:
 - **Are obtained by synchronisation**, then hiding, of locally visible actions and thus cannot be removed beforehand in the composed graphs
 - **Are not detected by persistent set methods**
- CCD is **less resource consuming** than on-the-fly τ -confluence elimination in the product graph
- CCD **can be combined** with compositional verification and persistent set methods



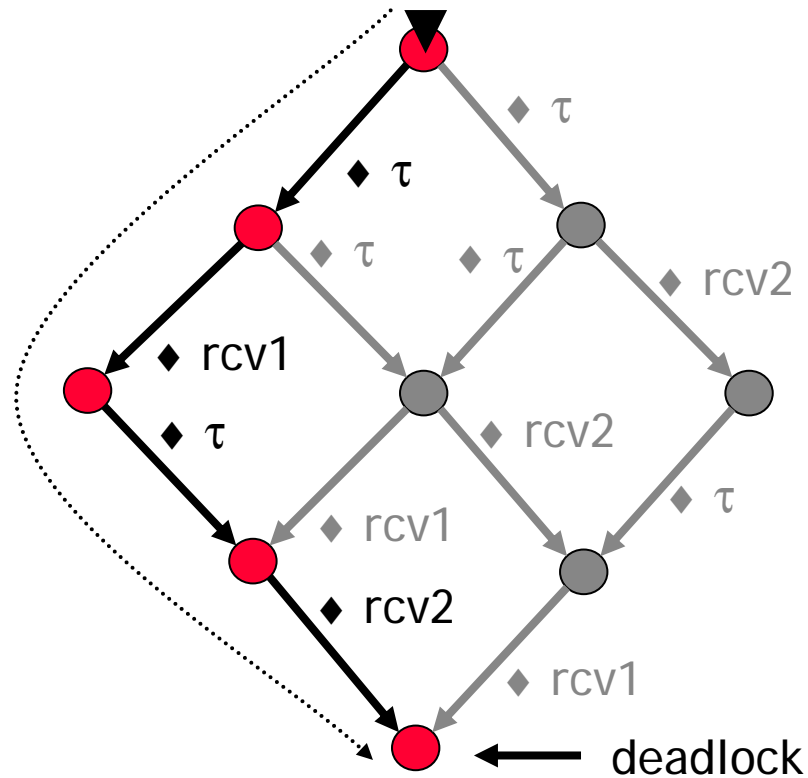
Confluence

- CCD requires a more general notion of *confluence*
 - Generalizes τ -confluence for visible actions
 - Is analogous to "confluent processes" (Milner) and lifted to transitions as Groote & van de Pol's τ -confluence
 - Has a strict and a non-strict variants
- A transition is [*strictly*] *confluent* ($\overset{\blacklozenge}{\longrightarrow}^a$) if:



Strict confluence theorem

- **Theorem:** Prioritization of strictly confluent transitions preserves deadlocks
- Formal proof available in INRIA RR-7078
- **Example:**

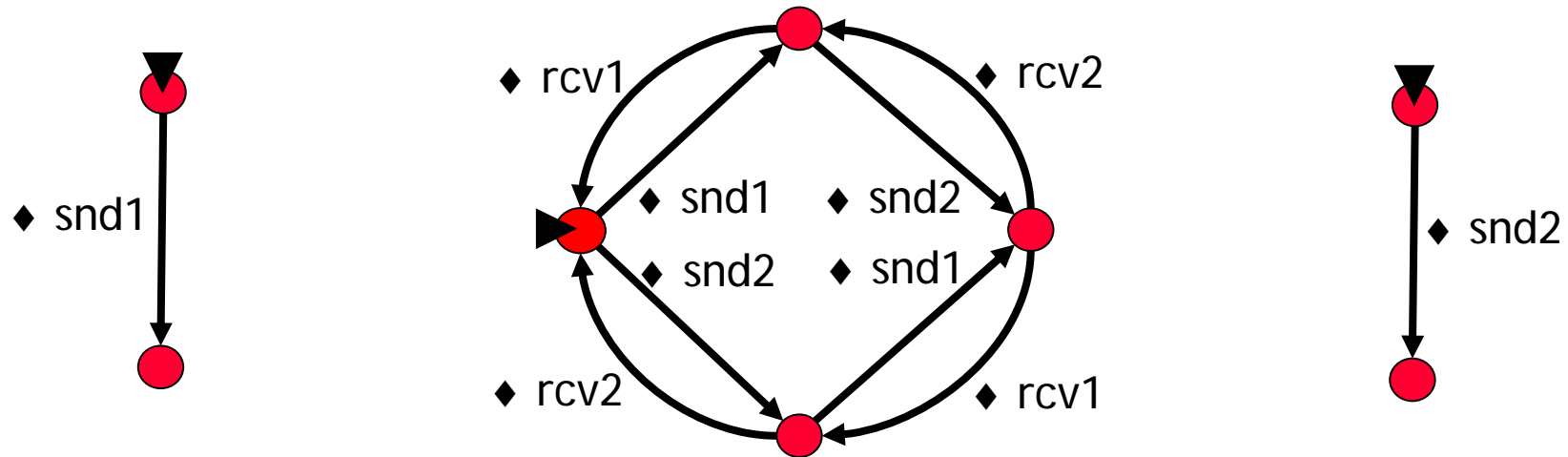


Compositional confluence theorem

- **Theorem:** Transitions obtained by synchronisation of [strictly] confluent transitions are [strictly] confluent
- Formal proof available in INRIA RR-7078
- **Corollaries:**
 - Prioritizing transitions obtained by synchronization of **strictly confluent** transitions preserves **deadlocks**
 - Prioritizing τ -transitions obtained by synchronization of **confluent** transitions preserves **branching equivalence**, as long as they do not close a circuit



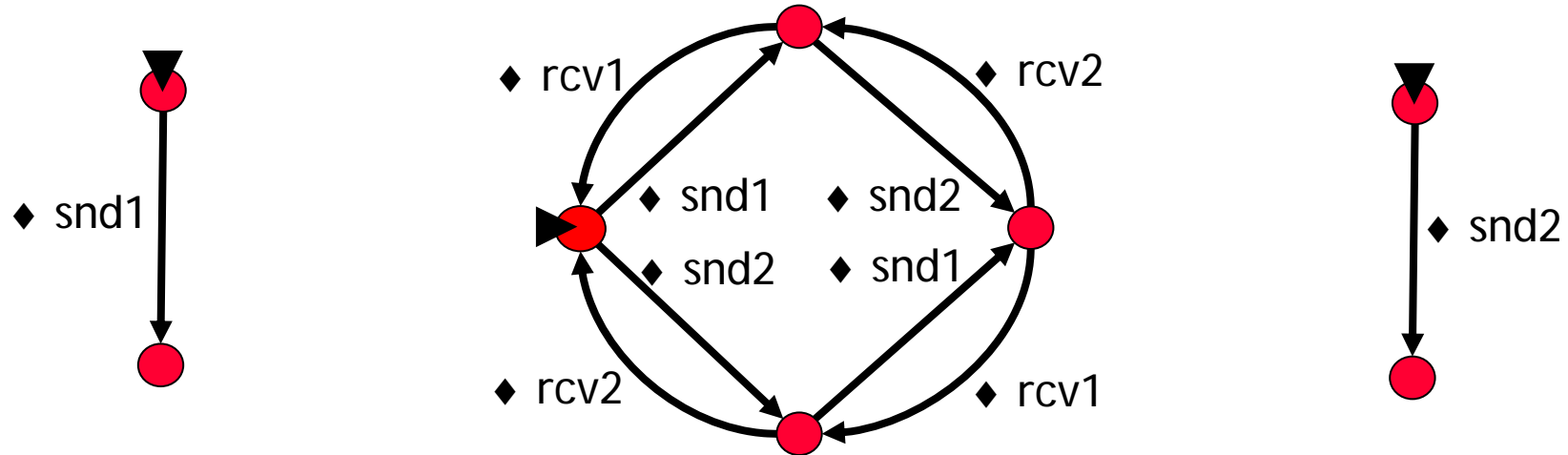
Example (1/2)



$(\text{snd1}, \text{snd1}, \bullet) \rightarrow \tau$ yields a τ -confluent transition in
init state as **both snd1-transitions are confluent**



Example (2/2)



- $S = \{(\text{snd1}, \text{snd1}, \bullet) \rightarrow \tau\}$ is **not persistent** in init state
 - S persistent if each operation $\notin S$ dependent on a operation $\in S$ cannot be enabled before an operation $\in S$ is fired
 - $((\bullet, \text{snd2}, \text{snd2}), \tau) \notin S$ dependent on $((\text{snd1}, \text{snd1}, \bullet), \tau) \in S$
 - Both rules are enabled in init state
- Same for $S = \{(\bullet, \text{snd2}, \text{snd2}) \rightarrow \tau\}$



Confluence detection

- Encode the problem as the resolution of a maximal fixed point **Boolean Equation System (BES)**:

$$\{ X_{s_1, a, s_2} =_{\forall} \wedge s_1 \rightarrow_b s_3 ($$

$$\begin{array}{l} \forall s_2 \rightarrow_a s_4 X_{s_3, a, s_4} \vee (b = \tau \wedge \forall s_3 \rightarrow_a s_2 \text{ true}) \\ \vee \\ (a = \tau \wedge s_3 = s_4) \end{array}$$

) }

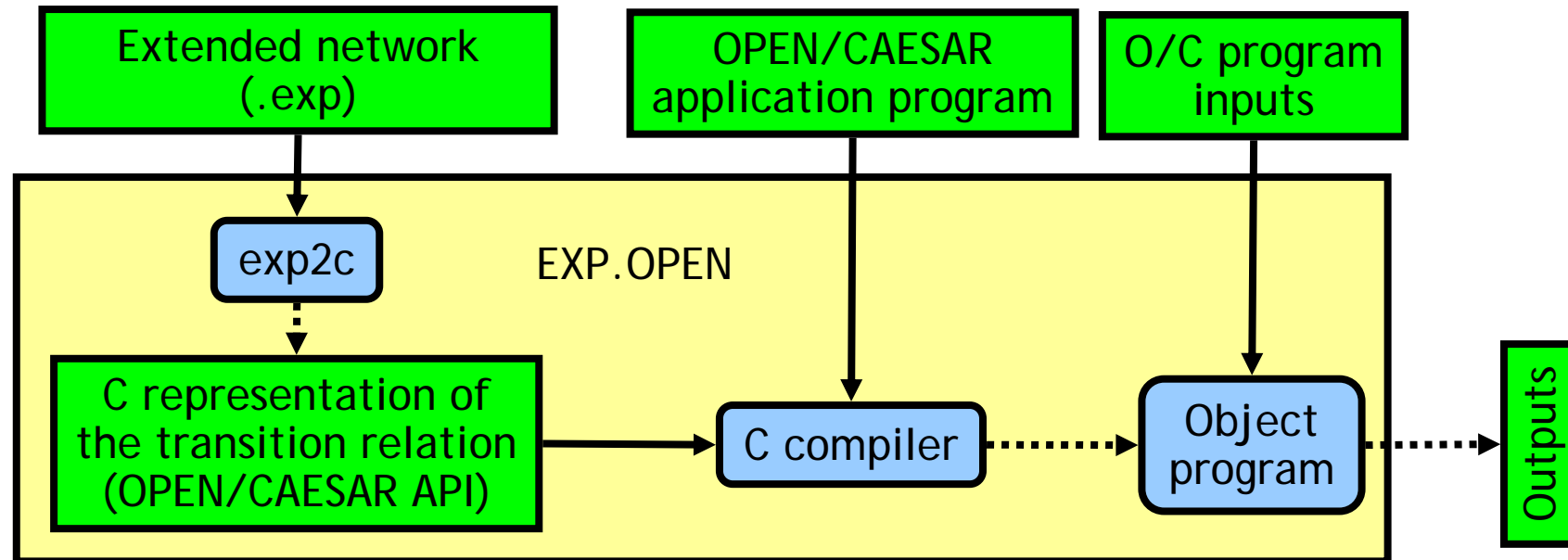
strict confluence

non-strict confluence

- $X_{s_1, a, s_2} \text{ true}$ iff $s_1 \rightarrow_a s_2$ confluent
- BES resolution carried out using a global linear-time algorithm [Andersen-94, Mateescu-00]



The EXP.OPEN 2.0 tool of CADP



- New option **-confluence**
 - Combined with persistent set methods (**-deadpreserving**, **-weaktrace**, or **-branching** options)
 - Search [strictly] confluent transitions in composed graphs
 - Use confluence information to prioritize transitions



Experimental results

branching (1/2)

- CADP demos available at <http://www.inrialpes.fr/vasy/cadp/demos>
- ODP (Open Distributed Processing) trader (demo 37)
 - **22 K st. / 158 K trans.** using compositional verification
 - no reduction using persistent sets
 - **0,5 K st. / 2,8 K trans.** using CCD
- Asynchronous circuit for Data Encryption (demo 38)
 - **1,4 K st. / 3,5 K trans.** using compositional verification
 - no reduction using persistent sets
 - **0,3 K st. / 0,6 K trans.** using CCD



Experimental results

branching (2/2)

- Examples provided by **ST Microelectronics** (critical part of a multiprocessor system on chip)
- ST example 1:
 - **5,4 M st. / 37,6 M trans.** using compositional verification
 - no reduction using persistent sets
 - **5,1 M st. / 24,7 M trans.** using persistent sets + CCD
- ST example 2:
 - **789 M st. / 8104 M trans.** using compositional verification
 - no reduction using persistent sets
 - **710 M st. / 6143 M trans.** using persistent sets + CCD



Experimental results

deadlocks

- ODP trader
 - 22 K st. / 158 K trans. using compositional verification
 - no reduction using persistent sets
 - 0,08 K st. / 0,1 K trans. using persistent sets + CCD
- ST example 1:
 - 5,4 M st. / 37,6 M trans. using compositional verification
 - 5,2 M st. / 34,2 M trans. using persistent sets
 - 0,39 M st. / 1,3 M trans. using persistent sets + CCD



Conclusion

- CCD (Compositional Confluence Detection) is **a new partial order reduction method**
 - It **works compositionally** by searching confluence in the composed graphs to detect confluence in the product
 - It can **improve the reductions obtained using persistent set methods**
- **CADP** (<http://www.inrialpes.fr/vasy/cadp>) supports CCD combined with **persistent sets, on-the fly verification** and **compositional verification**
- In the future, CCD could also be combined with **distributed graph generation**

