Kenneth J. Turner. The Formal Specification Language LOTOS: A Course For Users. Department of Computing Science and Mathematics, University of Stirling, Scotland, August 1989.

The Formal Specification Language

LOTOS

A Course for Users

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16th April 1996

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Preface

LOTOS is, of course, the product of a protracted standardisation process involving many people internationally. LOTOS has been defined by Sub-Group C of the joint ISO+IEC Sub-Committee JTC1/SC21/WG1, under the Chairmanship of Ed Brinksma (*University of Twente*, Netherlands).

The production of an early version of these lecture notes was supported by the CEC (through the Esprit SEDOS project), and by ICL (through the QUASARS project). I am grateful to Alastair Tocher (*International Computers*, UK) for his comments on the early version, and for the many students who have helped to debug them.

Chapter 1

Introduction

1.1 Scope and Objectives

- this course is intended to introduce all features of LOTOS
- the course conforms to the definition of LOTOS given in the International Standard (published in 1989)
- the course is intended for *users* of LOTOS, mainly those who wish to read and understand LOTOS specifications; the course therefore concentrates on developing an intuitive grasp of the language by presenting basic concepts, backed up by suggestive examples
- to the uninitiated, formal languages (and LOTOS is no exception) appear abstruse and impenetrable; it is hoped that readers who complete this course will have overcome their 'culture shock', and will feel able to pick up specifications in LOTOS and get something out of them
- it should be recognised, however, that LOTOS is a *formal* language; an intuitive understanding of the language is therefore not a substitute for understanding the implications of the formal semantics which underlie it
- this course is therefore only an introduction to LOTOS as defined in the ISO standard
- throughout the course, copious examples have been used; these have intentionally been kept light (and, at times, whimsical) in an attempt to keep the interest of the reader; it would be wrong to conclude that LOTOS was applicable to toy problems only
- the reader of the course is encouraged to try the examples; equally, the reader is encouraged to manufacture other examples which are relevant to day-to-day work; the reader should avoid trying to specify anything too complex at first

1.2 Background and Context

1.2.1 OSI

- ISO (*International Standardisation Organisation*) is producing Standards for OSI (*Open Systems Interconnection*) in response to the demands of manufacturers and users for compatible interconnection of data processing equipment
- OSI is a major 10-year effort, with thousands of man-years in design and specification to produce a rich variety of telecommunications standards for interconnection of heterogeneous equipment
- for the goal of compatible interconnection to be achieved, precise and clear specifications are needed; however, the vast majority of ISO specifications are currently in natural language
- there are major problems in achieving compatible interconnection:

  - how can ISO ensure that thousands of implementers world-wide will interpret OSI standards in a compatible way?
  - who will provide the definitive interpretation when the original developers of the standards have dispersed?

1.2.2 Formal Description Techniques

- the scale and complexity OSI led to the formation of a group to standardise formal specification languages for OSI
- ISO developed FDTs (*Formal Description Techniques*) to provide the basis for unambiguous interpretation of standards
- FDTs are required to help:
  - *specifiers* — precision, conciseness, clarity, verifiability
  - *implementers* — avoidance of over-specification, clear guidance as to what, not how
  - *testers* — isolation of implementation options, basis of rigorous testing against specifications
- work in ISO committee SC21/WG1 (*OSI Architecture*) was established in order to develop FDTs for OSI
- initially, around 20 different techniques were proposed, in two broad categories:
  - finite state-machine techniques, which led to ESTELLE (*Extended Finite State-Machine Language*)
  - algebraic techniques, which led to LOTOS (*Language Of Temporal Ordering Specification*)
- three FDT Sub-Groups were set up:
  - A - Architectural aspects of FDTs
  - B - ESTELLE
  - C - LOTOS
- a later collaboration on FDTs was set up with SGX/WP3 of CCITT (*International Telephone and Telegraph Consultative Committee*)
- CCITT have also standardised SDL (*Specification and Description Language*)
- the net result is that there are three officially-recognised FDTs in ISO and CCITT

1.2.3 Development of LOTOS

- Sub-group C initially adopted CCS (*Calculus of Communicating Systems*)
- some notation was later introduced from CSP (*Communicating Sequential Processes*)
- the development of LOTOS was based on architectural principles
- a prototype of the language was used on sample OSI Standards, leading to improvements in the language
- the ACT ONE data typing language was added to allow formal and abstract specification of data types
- LOTOS is the first piece of mathematics to be standardised internationally
- an International Standard (8807) for LOTOS was completed in August 1988, after 8 years' work!

1.2.4 The Nature of LOTOS

- LOTOS is a *specification language*, with a *formal* basis
- ESTELLE and SDL are *semi-formal implementation languages*, useful for describing reference implementations of OSI standards
- LOTOS is widely supported by academic institutions world-wide, and has significant industrial support
- LOTOS was designed for the specification of OSI systems, but is equally suitable for the specification of concurrent or distributed systems generally
- the formal basis of LOTOS ensures precision and analysability; however, this is bought at the price of needing special training in the language, and more brain- work to understand LOTOS specifications
- using LOTOS demands time and attention:
  - specification languages have a different philosophy from programming languages and natural languages
  - expressing oneself precisely requires discipline
  - thinking about what one really means is hard work

1.2.5 Application of LOTOS

- LOTOS has been used mainly on OSI, but this is not an intrinsic limitation
- LOTOS is applicable to sequential, concurrent, and distributed systems generally
- the following LOTOS specifications of OSI have been written (CL = *Connection-Less*, CO = *Connection-Oriented*)
  - **Application Layer** : FTAM (*File Transfer and Manipulation*) and ACSE (*Association Control Service Elements*)
  - **Presentation Layer** : some work (CO)
  - **Session Layer** : complete (CO)
  - **Transport Layer** : complete (CO)
  - **Network Layer** : Service (CL, CO) and Protocol (CL)
  - **Data Link Layer** : Service (CL, CO)
- LOTOS is being used on new OSI developments (e.g. ODP (*Open Distributed Processing*) and OSI Management)

1.2.6 Related Formal Languages

- LOTOS is an algebraic specification language, which includes abstract data type languages such as ACT ONE, and process algebras such as CSP and CCS
- notable features of LOTOS are:
  - its integrated data typing and behavioural modelling
  - its handling of non-determinism and concurrency
  - its orientation towards OSI
- LOTOS data typing is similar to other languages such as CLEAR and OBJ
- LOTOS uses CSP-like notation for events, '!' and '?', but these are more than just 'input' and 'output'
- this gives LOTOS powerful synchronisation features not in CCS or CSP (e.g. multi-way synchronisation or value negotiation)

- LOTOS gates correspond to CSP channels; LOTOS processes interact synchronously, just as in CSP and CCS
- in LOTOS, unlike in CCS and CSP, it is *gates* not whole *events* which are hidden, passed as parameters, etc.
- LOTOS has a 'kind choice' operation (CSP '[ ]' or CCS '+')
- LOTOS does not have the CSP non-deterministic choice operation '⊓'
- for non-determinism, LOTOS follows the CCS approach of an internal event (CCS 'ø')
- LOTOS has enabling and disabling operators in addition to basic CCS or CSP ones
- LOTOS has CSP parallel composition, which forces synchronisation: there is no need to restrict and hide events for synchronisation, as in CCS

1.3 Basics of Process Algebra

1.3.1 Processes

- a *process* is a component of a specification; it is the abstraction of an activity in an implementation, and communicates with other processes
- a process is considered to be a black-box at some level of abstraction; only the external behaviour of a process is considered
- processes share a communication mechanism called an *interaction point*, as shown in Figure 1.1; this is the abstraction of an interface in an implementation

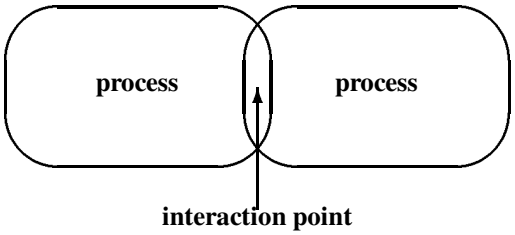


Figure 1.1: Interaction Point

- in LOTOS, the specification concept of interaction point corresponds to the language concept of *event gate* (or just *gate*)
- processes are specified by giving a *behaviour expression* which defines their externally visible behaviour in terms of the permissible sequences of *events* in which they may participate
- in these course notes, processes are given upper-case names such as:

ACTIVATE_ ALARM DATA_ TRANSFER

1.3.2 Events

- an *event* represents a *synchronisation* between processes
- an *event offer* represents the ability of one process to participate in an event; events external to a process are resolved in conjunction with the *environment* of the process
- an event normally requires the participation of two or more processes; three kinds of event are possible:

  - *pure synchronisation* - no values are exchanged between the processes
  - *value establishment* - one or more processes supply a specific value which lies in the set acceptable to the other process(es)
  - *value negotiation* - two or more processes agree on a set of values

- events are considered to be *atomic*, i.e. at the level of abstraction of the specification, the synchronisation of the processes and the associated information are established at the same time; in an implementation, an event may of course correspond to a sequence of elementary steps
- the set of events in which a process can potentially participate is called the *alphabet* of the process
- the set of events in which a process can immediately participate at any point in the unfolding of its behaviour is called the *initials* of the process
- in the earlier parts of these notes, events with only an event gate are considered; more complex structures for events are considered later
- a behaviour expression is evaluated in an environment which offers it events to synchronise on; the environment may be another behaviour expression which it is combined with, or may be something external
- if synchronisation on event offers from the environment is not possible, *deadlock* has occurred (cf. a deadly embrace or a terminated process)
- if no events are offered by a behaviour expression, *livelock* has occurred (cf. looping)
- in these course notes, events are given lower-case names such as:

  timer_ expired **ready**

1.3.3 Temporal Ordering

- specifications in LOTOS give the *temporal ordering* of events, i.e. the *relative* ordering of events in time
- LOTOS abstracts away from absolute timing considerations, e.g. that an event must occur at a specific time or after a specific period
- LOTOS specifications use operators which combine behaviour expressions to yield more complex behaviour expressions; these operators obey well-defined laws which enable any specification to be interpreted unambiguously
- the data typing part of LOTOS is given semantics using the theory of algebras; LOTOS operators for combining behaviour expressions can be shown to satisfy algebraic laws which characterise them
- a convenient diagrammatic notation for thinking about behaviour in languages like LOTOS is the *synchronisation tree*, as shown in Figure 1.2

occurrence of an event e

choice between events e_1 or e_2

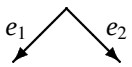


Figure 1.2: Synchronisation Tree

1.4 Background Needed

1.4.1 Differences from Programming

- surprisingly little technical knowledge is needed to achieve some familiarity with LOTOS a numerate background, with a small amount of basic mathematics and computing, is probably sufficient
- a knowledge of programming languages will also help, but beware that LOTOS being a *specification* language, treats some concepts in a fundamentally different way
- *variables* in LOTOS are mathematical variables: they are simply names which happen to be bound to a particular value in particular, one does not assign a value to a LOTOS variable as one might assign a value to a store location; in LOTOS one cannot write, for example:

  $$x := x + 1$$

  one must in effect define a new variable (x', say) which is bound to the value of the old x plus 1
- *recursion* in programming languages is usually thought of in terms of a stack which records return addresses; a well-behaved program unwinds this stack before continuing
- in LOTOS, however, one may exit from within a recursive piece of behaviour to do something completely different; this is because recursion in LOTOS is equivalent to replacing a recursive call with a copy of the text defining the behaviour
- *efficiency* is an important consideration in programming, but is not appropriate in LOTOS; one should not think of 'implementing' LOTOS literally, and must therefore not be concerned about matters of efficiency
- a good specification clearly defines *what* and not *how*; this contrasts sharply with programming, where one frequently makes design decisions to optimise the use of store, processor, etc.
- *over-specification* must be carefully avoided in LOTOS; one must always concentrate on external requirements and black-box behaviour, not structures or algorithms to implement these
- for example, one should specify a queue in terms of the operations on it (enqueue, dequeue, head, etc.) rather than in terms of linked-lists, algorithms for skipping down the queue, etc.

1.4.2 Sets and Relations

- a *set* can be though of as an unordered collection of elements in which duplicates are ignored (this glosses over certain mathematical niceties)

  sets are usually written in curly brackets, for example:

  $$\{Kenneth; John; Turner\}$$

- the *empty set*, $\{\}$, has no elements
- each element in a set is a *member* of that set; one set is *included* in another if its members are also members of the other set; the first such set is a *subset* of the second
- two sets are equal if they have the same members (i.e. each is a subset of the other)
- the *cartesian product* of a number of sets is the set of all *tuples* (ordered lists) drawn from the sets; the symbol $\times$ is used for this 'multiplication' operation, for example:

  $$\{run; fall\} \times \{down; off; out\}$$

  is:

  $$\{ < run; down >; < run; off >; < run; out >; < fall; down >;$$
  $$< fall; off >; < fall; out > \}$$

- the *powerset*, $\mathcal{P}$, of a set is the set of all its subsets; for example:

$$\mathcal{P}\{p; i; g\}$$

is:

$$\{\{\}; \{p\}; \{i\}; \{g\}; \{p; i\}; \{p; g\}; \{i; g\}; \{p; i; g\}\}$$

- a *relation* between two sets is a set of ordered pairs of elements from the two sets (it is a subset of their cartesian product); for example, Figure 1.3 corresponds to:

$$\{< lemon; yellow >; < lime; yellow >; < lime; green >; < apple; green >;$$

$$< apple; red >; < longan; stramineous >\}$$

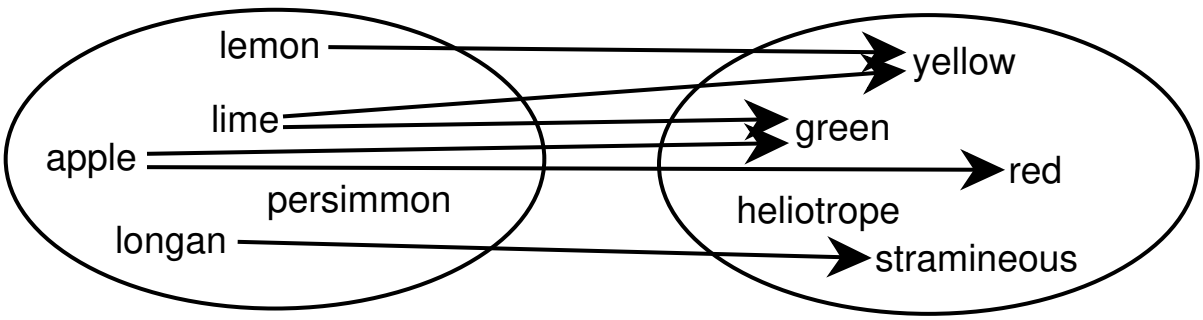


Figure 1.3: Example of a Relation

1.4.3 Functions

- a *function* is a relation which maps an element in one set to exactly one element in another set (several elements may get mapped onto the same element)
- the set to which the function is applied is its *domain*; the the set to which the function maps is its *codomain*; the following mathematical notation is used for this:

$$F : D \rightarrow C$$

where F is the function, D is its domain, and C is its codomain

- a function may also map from several sets to several sets; for example, in integer arithmetic:

$$divide : dividend \times divisor \rightarrow quotient \times remainder$$

which has as 'arguments' the number to be divided and the divisor itself, and has as 'results' the quotient and remainder

- however, functions in LOTOS are allowed to have only one named set as codomain; one solution to this is to define separate functions for each codomain; for example:

$$divquot : dividend \times divisor \rightarrow quotient$$

$$divrem : dividend \times divisor \rightarrow remainder$$

such functions are called *operations* in LOTOS

- a *total function* or *operation* gives a mapping for each element in its domain, otherwise it is *partial*
- an *associative* operation is one in which the bracketing is irrelevant; for example, in ordinary arithmetic:

$$(a \times b) \times c = a \times (b \times c)$$

  and so may be written without ambiguity as:

$$a \times b \times c$$

- a *commutative* operation is one in which the order of the operands does not matter; for example, in ordinary arithmetic:

$$a \times b = b \times a$$

1.4.4 Syntax and Semantics

- syntax is usually expressed using a *grammar*; this consists of a set of rules in which *non-terminal symbols* (variables of the grammar) are defined in terms of non-terminal symbols and *terminal symbols* (constants of the grammar)
- the definition of LOTOS syntax includes the following constructs:
  - recursion is expressed by, for example:
    echo = "hello" echo
    which defines **echo** to be **hello** (a terminal symbol) followed by **echo**; in other words, **echo** is an unbounded repetition of **hello**
  - a choice is expressed by, for example:
    mood = happy | sad
  - an optional part of a rule is expressed by, for example:
    sentence = subject verb [object]
    which says that zero or one occurrences of **object** are allowed
- a set of *derivation rules* is a system of logic for dealing with inferences; such a system is used to express the semantics of LOTOS operators
- a derivation system has a set of *axioms* (logical formulae from which others are derived), and a set of *assertions* (logical formulae which may or may not be derivable)
- a logical formula is derived by applying *inference rules* of the form:

$$\frac{P_1;\ :::\ ;\ P_n}{Q}$$

  meaning that, given P_1 up to P_n, Q may be derived
- the shorthand notation:

$$\vdash P$$

  means that P can be derived from the axioms and the inference rules

1.5 Summary

- LOTOS is a *formal specification language* being developed by ISO for the specification of OSI Standards
- LOTOS is generally applicable to the specification of *concurrent* and *distributed systems* generally
- LOTOS is an *algebraic specification language* which describes systems by giving their externally visible behaviour in terms of the permissible sequences of events they may participate in
- LOTOS specifications consist of *behaviour expressions* (often generalised as *processes*) which synchronise on *events* at *gates*
- events may be *pure synchronisation*, *value establishment*, or *value negotiation*
- the *alphabet* of a behaviour expression is the set of all its potential events, whereas its *initials* are those it can immediately offer

Chapter 2

Basic Process Expressions

2.1 Sequence

- the *sequential composition operator* ';' is used to prefix a behaviour expression with an event called an *action prefix*; for example:

  button_pressed; RING_BELL

- action prefixes associate to the right; for example:

  connect_request; connect_confirm; data; DISCONNECT

  means:

  connect_request; (connect_confirm; (data; DISCONNECT))

- the alphabet of this behaviour expression is:

  $$\{connect_request; connect_confirm; data\}$$

  plus whatever events there are in DISCONNECT

2.2 Choice

- the *choice operator* '[]' is used when alternative behaviours are allowed; for example:

  (lift_arrived; ENTER) [] (lift_broken; USE_STAIRS)

- choice is associative (as one would expect); for example:

  EAT [] DRINK [] BE_MERRY

means the same as:

(EAT [] DRINK) [] BE_ MERRY

and:

EAT [] (DRINK [] BE_ MERRY)

but notice that one cannot eat, drink, *and* be merry!

- choice is also commutative (as one would again expect); for example:

  PAY_ A_ FINE [] TAKE_ A_ CHANCE

  means the same as:

  TAKE_ A_ CHANCE [] PAY_ A_ FINE

- the initials of the behaviour expression:

  (lunch_ bell; EAT) [] (boss_ here; WORK) [] (fire; PANIC)

  are:

  $$\{lunch_bell; boss_here; fire\}$$

- in the straightforward case, the choice between alternatives is resolved by the environment of the process; in the example above, if the environment offers only **fire** then the process will PANIC
- more complicated cases involving non-determinism are considered later in these notes

2.3 Parallelism

2.3.1 Interleaving

- the *interleaving parallel composition operator* '|||' is used to allow behaviours to unfold completely independently in parallel; the events from each behaviour expression are interleaved
  for example:

  (data_ in; data_ out; BUFFER) ||| (read; mark; digest; BOOK)

  includes the following behaviours:

  $$data_in; read; mark; data_out; digest:::$$

  $$read; mark; digest; data_in:::$$

- '|||' is associative and commutative (as one would expect, in order to capture the intuitive concept of running in parallel)

2.3.2 Synchronisation

- the *synchronising parallel composition operator* '||' is used where there are events that need to be synchronised; in this case the events (strictly, gates) which occur in *either* of the behaviour expressions are obliged to synchronise

  for example:

  (bang; start; finish; ATHLETE)
  ||
  (bang; start; finish; STARTER)

  may engage in the following sequence of events:

  bang; start; finish; :::

- if only certain events are to be synchronised, the '|[:::]|' form of the operator is used, with the events (strictly, gates) named between '[' and ']'; for example:

  (off_hook; dial; answer; speak; on_hook; TELEPHONE)
  |[dial]|
  (find_number; dial; engage_brain; speak; CALL)

  will synchronise only on the **dial** event, and will allow **speak** in the second behaviour expression before **answer** and after **on_hook** in the first behaviour expression (a possibly realistic situation)

- '|[:::]|' with an explicit empty list, or with no gates which occur in the two behaviour expressions, is equivalent to '|||'
- '|[:::]|' with an explicit list which is in fact the union of the two sets of gates is equivalent to '||'
- like '|||', '||' or '|[:::]|' is associative and commutative
- parallel composition can be used to express independent constraints; for example, the constraints 'breakfast must precede lunch' and 'lunch must precede dinner' can be expressed by:

  (breakfast; lunch; AM) |[lunch]| (lunch; dinner; PM)

  this allows a separation of concerns, not to mention eating habits!

2.4 Termination

2.4.1 Inaction and Success

- the simplest behaviour expression is '**stop**', which offers no events and therefore does nothing; it is used to represent *inaction* or *deadlock*
- *successful termination* of a behaviour expression is represented by '**exit**', which offers a special event ¡ and then behaves as **stop**
- ¡ is part of the underlying mathematical model of LOTOS and is *not* an event which can be explicitly offered; ¡ may also be an implied initial of a behaviour expression

- successful termination of a sequence (using the ';' operator) depends on whether the right-most behaviour expression is **stop** or **exit**

  for example:

  clock_in; clock_out; **exit**

  may terminate successfully, but:

  born; died; **stop**

  can not

- successful termination of a choice (using the '[]' operator) depends on the successful termination of *one* of the behaviour expressions; for example:

  (fail; **stop**) [] (catastrophe; **stop**)

  can not terminate successfully, but:

  (lift_off; **exit**) [] (armageddon; **stop**)

  may succeed

- successful termination of a parallel composition (using the '|||', '||', or '|[:::]|' operators) depends on the successful termination of *all* of the behaviour expressions; for all these operators, the special termination event ¡ is always synchronised — even in the case of '|||'

  for example:

  (finished; **exit**) ||| (done; **exit**)

  may terminate successfully

2.4.2 Enabling

- as a generalisation of the ';' operator, which is used for the sequential composition of an *event* and a *behaviour expression*, the '>>' operator (pronounced *enables*) combines two *behaviour expressions* in sequence

  for example, if process SHOP is:

  visit_shop; buy_food; come_home; **exit**

  and process EAT is:

cook_ food; eat_ food; **stop**

then the process DINE, defined by:

SHOP >> EAT

denotes the possible sequence of events:

visit_ shop; buy_ food; come_ home; cook_ food; eat_ food

- **exit** (i.e. the ¡ event) is absorbed by a following '>>'
- if the left-hand behaviour expression does not terminate successfully, the right-hand behaviour expression will not apply; for example:

  (prime_ 5; prime_ 7; (prime_ 9; **stop** [] **exit**)) >> (prime_ 11; **exit**)

  can successfully terminate after the sequence of events:

  prime_ 5; prime_ 7; prime_ 11

  but will deadlock after the sequence of events:

  prime_ 5; prime_ 7; prime_ 9

- '>>' is associative, but is not of course commutative

2.4.3 Disabling

- a frequent occurrence in specifications is the need to specify behaviour which may be interrupted by something else (e.g. disconnection may terminate data transfer)
- the '[>' operator (pronounced *disabled by*) allows the right-hand behaviour expression to interrupt the left-hand behaviour expression; if this happens, the future behaviour is that of the right-hand behaviour expression only
- if the left-hand behaviour expression terminates successfully, then the combination also terminates successfully (i.e. disabling is no longer possible); this is to allow for contingencies

  for example:

  (send_ data; reset_ timer; receive_ acknowledgement; **exit**)
  [>
  (timer_ expired; sound_ alarm; **stop**)

  may terminate successfully if an acknowledgement is received to a message, but may sound an alarm if no acknowledgement is received within some time period

- beware that behaviour may be disabled between its 'last' event and **stop** or **exit**

  thus, in the example above, **timer_ expired** may occur *before* **reset_ timer** and *after* **receive_ acknowledgement**; this could be realistic in an actual implementation

- the left-hand behaviour expression is often non-terminating (iterative or recursive), and the right-hand one is often a closing-down behaviour expression; for example:

  DATA_ TRANSFER [> DISCONNECTION

- '[>' is associative, but is not of course commutative

2.5 Non-Determinism

2.5.1 Non-Determinism due to Choice

- the events in a process specification are *event offers*; the actual events which happen may be influenced by the environment of the process

  for example:

  (wake_ up; MAKE_ COFFEE) [] (open_ wine; GET_ DRUNK)

  will result in MAKE_ COFFEE or GET_ DRUNK depending on the sobriety of the environment

- however, identical events may be offered as alternatives: in this case, the environment *cannot* influence which branch is taken; the choice is made non-deterministically

  for example:

  (eat_ out; CHINESE_ MEAL) [] (eat_ out; INDIAN_ MEAL)

  will result in CHINESE_ MEAL or INDIAN_ MEAL after the environment has offered **eat_ out**, and cannot be influenced

- non-determinism is best understood by considering the synchronisation tree for the behaviour; for example:

  (a; b; **stop**) [] (a; c; **stop**)

  has the synchronisation tree shown in Figure 2.1

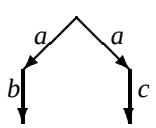


Figure 2.1: Synchronisation Tree with Initial Choice

but:

a; ((b; **stop**) [] (c; **stop**))

has the synchronisation tree shown in Figure 2.2

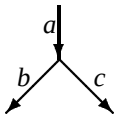


Figure 2.2: Synchronisation Tree with Deferred Choice

- in the first case, the environment offers **a** but has no choice as to what follows: it may be offered **b** or **c**; in the second case, the environment can still decide on **b** or **c** after offering **a**

2.5.2 Hiding

- LOTOS supports top-down decomposition of behaviour: processes (specification components) can be progressively decomposed into simpler processes
- however, it is important that the details of this decomposition (in particular, any internal events) are not visible at a higher level
- the *hiding operator* '**hide** ... **in**' is therefore used to hide events (strictly, gates) which are internal to the behaviour of a system; for example:

  (begin; middle; **exit**) |[middle]| (middle; end; **exit**)

  may engage in the sequence of events:

  begin; middle; end; ¡

  but:

  hide middle **in** ((begin; middle; **exit**) |[middle]| (middle; end; **exit**))

  may engage in only:

  begin; end; ¡

2.5.3 Internal Events

- a hidden event is represented by **i**, the internal event; **i** may be an initial of a behaviour expression

  for example:

  hide second **in** (first; second; third; **stop**)

may engage in:

first; **i***; third*

- the internal event represents non-determinism since the environment may not influence it; because the internal event may affect subsequent behaviour due to an internal choice it is useful to represent it because it 'explains' why a particular behaviour happened

  for example:

  WORK [] (**i**; GO_ TO_ BED)

  indicates that GO_ TO_ BED may happen without WORK being an option; the **i** represents a decision which cannot be influenced by the environment

  this might arise from:

  hide dawn_ chorus **in** (WORK [] (dawn_ chorus; GO_ TO_ BED))

- once the internal event has occurred, only the behaviour following it is allowed
- it is quite legitimate for all branches to be 'protected' with an **i**; for example:

  (**i**; WORK) [] (**i**; STRIKE) [] (**i**; GO_ ON_ HOLIDAY)

- the internal event is one way of expressing implementation freedom (where there are different ways to do something), or some internal decision (which it is not appropriate to spell out at a given level of abstraction)
- internal events are indistinguishable from each other
- when **exit** combines with '>>', the result is that the ¡ event turns into an internal event

2.5.4 Behaviour Equivalence

- the introduction of the internal event raises questions of when two behaviour expressions mean the same thing
- two behaviour expressions are *isomorphic* if they can be transformed into each other by systematic renaming; for example:

  (slithy; slithy; mimsy; BOROGOVE)
  []
  (mimsy; slithy; BOROGOVE)

  is isomorphic to:

  (lubricilleux; lubricilleux; enmime; GOUGEBOSQUET)
  []
  (enmime; lubricilleux; GOUGEBOSQUET)

- two behaviour expressions are *observationally equivalent* if they exhibit the same behaviour ignoring the internal events; under this equivalence, any *finite* sequence of internal events can be ignored except in the context of a choice

  for example:

  aye; **i**; **i**; EYE

  is observationally equivalent to:

  aye; EYE

  for example:

  (a; **i**; b; **exit**) >> ((c; **stop**) [] (d; **i**; **exit**))

  is observationally equivalent to:

  a; b; ((c; **stop**) [] (d; **exit**))

- an infinite sequence of internal events is called *infinite chatter* and corresponds to *livelock*; for example, a process LIVELOCK may be defined as:

  hide beep **in** (beep; LIVELOCK)

- *observational congruence* holds when one behaviour expression can be substituted for another in *all* contexts; for example:

  i; PROC1

  is not observationally congruent to:

  PROC1

  because it changes the overall meaning when substituted for it in a context such as:

  PROC1 [] PROC2

2.6 Behaviour Definitions

2.6.1 Definition and Instantiation

- a process is given a name and parameters by a *behaviour definition*; for example:

  process GIVE_ LECTURE
  [write_ notes, give_ lecture, understand_ subject] : **exit** :=

  write_ notes; give_ lecture; understand_ subject; **exit**

  endproc

- a process may optionally be followed by a list of events (strictly, gates) inside '[...]'
- the keyword **exit** is used in the heading line if the process may terminate successfully; **noexit** is used if it can never terminate
- the behaviour definition gives *formal parameters* to the process; the process is referred to in a *process instantiation* by giving *actual parameters* such as:

  GIVE_ LECTURE
  [plagiarise_ notes_ by_ Ed , lecture_ at_ Stirling, understand_ LOTOS]

- a process may also be optionally defined to have parameters and results; for example:

  process PROC [gates] (parameters) : **exit** (results) :=

  ...

  endproc

  this is covered in detail later in these course notes

2.6.2 Recursion

- a process may refer to itself in its definition; for example:

  process PRESS_ UP [up, down] : **noexit** :=

  up; down; PRESS_ UP [up, down]

  endproc

- it is important not to think of this kind of recursion in a programming language sense: there is no 'subroutine stack', which has to be unwound neatly; recursion is equivalent to repeated textual substitution of the definition of a process

- (tail) recursion is the way of expressing *iteration* in LOTOS
- mutually recursive processes are also allowed; for example:

```
process FLIP [in_0, in_1, out_0, out_1] : noexit :=

   in_1; FLOP [in_0, in_1, out_0, out_1]
[]
   out_0; FLIP [in_0, in_1, out_0, out_1]

endproc

process FLOP [in_0, in_1, out_0, out_1] : noexit :=

   in_0; FLIP [in_0, in_1, out_0, out_1]
[]
   out_1; FLOP [in_0, in_1, out_0, out_1]

endproc
```

2.7 Summary

- basic LOTOS operators are:

sequence	event; NEXT_PROCESS
enabling	FIRST_PROCESS >> SECOND_PROCESS
choice	POSSIBILITY [] POTENTIALITY
parallelism	INTER ||| LEAVED
	EITHER || ALPHABET
	EXPLICIT |[subset]| ALPHABET
disabling	HEALTH [> ACCIDENT
hiding	**hide** top_secret **in** MI5
termination	**stop**
	exit

- the possibility of successful termination depends on the success of *one* process for the choice operator, and *all* processes for the parallel operators
- *non-determinism* is expressed when the environment is offered a choice of the same events, or with **i** (which may arise explicitly or due to hiding)
- *observation equivalence* holds when two processes have the same behaviour, ignoring (finite sequences of) internal events
- *behaviour definitions* give processes names and parameters

```
process VOID [null] : noexit :=

   null; stop

endproc
```

2.8 Exercises

2.8.1 Synchronisation Tree

- consider letters to correspond to events, and words to correspond to sequences of events
- construct a synchronisation tree which spells the words ‘spa’, ‘stop’, ‘strap’, and ‘strop’
- translate this into LOTOS

2.8.2 One-Shot Dictionary

- specify the following application, which has been developed to translate English words into both French and German
- unfortunately, owing to shortage of development funds, the only words which are currently translated are ‘cat’ (‘chat’, ‘Katze’) and ‘dog’ (‘chien’, ‘Hund’)
- even more unfortunately, due to severe overloading of the CPU the system manager has decreed that a user may have only one word translated each time the application is used; after this, the application terminates successfully
- however, when the application was built it was designed with parallel processing in mind; the French and German translation functions therefore run concurrently
- finally, the user has option of killing the entire application at any time, resulting in the application aborting

2.8.3 Events

- what is the alphabet of the behaviour expression:

```
  BUCKET_BRIGADE [from, to]
[>
  BUCKET_KICKED [done]
```

given the definitions:

```
process BUCKET_BRIGADE [input, output] : noexit :=

  input;
  output;
  BUCKET_BRIGADE [input, output]

endproc

process BUCKET_KICKED [dead] : noexit :=

  dead;
  stop

endproc
```

- after the first event of the behaviour expression:

UNGAINLY_ QUICKSTEP [lento, rapido, errore]

what are its initials? UNGAINLY_ QUICKSTEP is defined by:

```
process UNGAINLY_ QUICKSTEP [slow, quick, stumble] : noexit :=

   slow;
   (
      slow; quick; quick; exit
   []
      i; stumble; exit
   )
>>
   UNGAINLY_ QUICKSTEP [slow, quick, stumble]

endproc
```

- give a behaviour expression without internal events which is observationally equivalent to:

  I_ dont_ see; **i**; to; **i**; with_ him; **stop**

- give a behaviour expression without internal events which is observationally equivalent to:

  hide a **in** ((**i**; a; B [a]) || (a; C [a]))

- give a behaviour expression without internal events which is observationally equivalent to:

```
   (Paris; DANS) ||| (le; printemps; exit)
[]
   hide le in ( (le; stop) |[le]| (le; printemps; exit) )
```

2.8.4 LOTOS-Speak

- it is reported that ISO has instructed all national standards organisations to install LOTOS-Speak machines in their headquarters
- these machines accept a 10 ECU coin and then utter a random sentence allowed by the following grammar (see section 1.4.4 for the notation):

```
sentence          =   noun_phrase verb_phrase adjective_phrase .
noun_phrase       =   "LOTOS" ["syntax"] .
verb_phrase       =   "seems" | "proves" .
adjective_phrase  =   adverb adjective .
adverb            =   "really" | "unbelievably" .
adjective         =   "awful" | "wonderful" .
```

- after speaking, the machine is willing to accept a new coin and repeat the above
- unfortunately, the machine was not designed from a formal specification and is liable to fail permanently, issuing **aargh** once
- specify processes whose permissible sequences of events correspond to the sentences these machines can utter

2.9 Possible Solutions

2.9.1 Synchronisation Tree

- the tree shown in Figure 2.3 may be specified as:

```
s;
(
   (p; a; stop)
[]
   (t;
      (
         (o; p; stop)
      []
         (r;
            (
               (a; p; stop)
            []
               (o; p; stop)
            )
         )
      )
   )
)
```

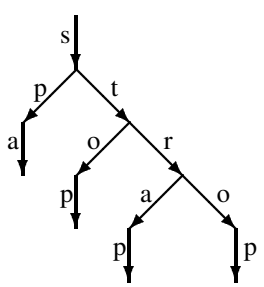


Figure 2.3: Possible Synchronisation Tree

2.9.2 One-Shot Dictionary

- a possible solution is:

```
(
   (
```

```
            (cat; chat; exit)
          []
            (dog; chien; exit)
          )
        |[cat, dog]|
          (
            (cat; Katze; exit)
          []
            (dog; Hund; exit)
          )
        )
      [>
        (kill; stop)
```

- this is divided up by language (French translation and German translation), but could also be divided up by word ('cat' translation and 'dog' translation)

2.9.3 Events

- from, to, done
- lento, **i**
- I_dont_see; to; with_him; **stop**
- **hide** a **in** (B [a] || C [a])
- ((Paris; DANS) ||| (le; printemps; **exit**)) [] (printemps; **stop**)

2.9.4 LOTOS-Speak

```
process MACHINE
  [ten_ECU, LOTOS, syntax, seems, proves, really, unbelievably, awful,
    wonderful, aargh] : noexit :=

  SENTENCE
    [ten_ECU, LOTOS, syntax, seems, proves, really, unbelievably, awful,
      wonderful]
[>
  FAIL [aargh]

endproc

process SENTENCE
  [ten_ECU, LOTOS, syntax, seems, proves, really, unbelievably, awful,
    wonderful] : noexit :=

  ten_ECU;
  NOUN_PHRASE [LOTOS, syntax]
>>
  VERB_PHRASE [seems, proves]
>>
  ADJECTIVE_PHRASE [really, unbelievably, awful, wonderful]
>>
  SENTENCE
```

```
    [ten_ECU, LOTOS, syntax, seems, proves, really, unbelievably, awful,
      wonderful]

endproc

process NOUN_PHRASE [LOTOS, syntax] : exit :=

  LOTOS; ( (i; syntax; exit) [] exit)

endproc

process VERB_PHRASE [seems, proves] : exit :=

  (i; seems; exit) [] (i; proves; exit)

endproc

process ADJECTIVE_PHRASE
  [really, unbelievably, awful, wonderful] : exit :=

  ADVERB [really, unbelievably]
>>
  ADJECTIVE [awful, wonderful]

endproc

process ADVERB [really, unbelievably] : exit :=

  (i; really; exit) [] (i; unbelievably; exit)

endproc

process ADJECTIVE [awful, wonderful] : exit :=

  (i; awful; exit) [] (i; wonderful; exit)

endproc

process FAIL [aargh] : noexit :=

  i; aargh; stop

endproc
```

Chapter 3

Basic Value Expressions

3.1 Values

- so far, events and event gates have been treated as synonymous; process parameters and results have also not been covered
- LOTOS contains a data-typing sub-language called ACT ONE which allows variables and value expressions such as:

  Number_ of_ Hackers

  succ (succ (succ (0)))

  Concurrency & Synchronisation

- in these course notes, variables names are given initial capitals such as:

  Repeat_ Count **Data_ Request**

- LOTOS variables are *mathematical variables*: they are not assigned values like a store location; they represent the binding of a name to a value
- an *expression* consists of *constants*, *variables*, and *operations*
- a variable is given a value in a *binding occurrence*; its *scope* is generally the behaviour expression which follows its binding occurrence
- a *sort* is a set of values; the following are common sorts:

Bool_ Sort	the Booleans (*true*, *false*)
Int_ Sort	the integers (..., *-1*, *0*, *+1*, ...)
Nat0_ Sort	the non-negative integers (*0*, *1*, ...)
Nat_ Sort	the positive integers (*1*, *2*, ...)

- a variable is given a sort when it is bound, for example:

Bank_ balance : Nat_ Sort

Liquorice : All_ Sort

3.2 Local Definitions

3.2.1 Local Variables

- local variables may be defined using the *local definition operator* '**let** ... **in**'; for example:

```
let E : Energy_ Sort = M * SQR (C) in SPECIAL_ RELATIVITY

let Salary : Nat_ Sort = 50000, Blood_pressure : Nat_ Sort = 140 in JOB
```

- as usual, the sort of a local variable is given in these local definitions following ':'
- the scope of such local variables is the behaviour expression which follows **in**
- the closest binding determines which value is denoted; for example, assuming decimal notation and arithmetic in the following:

```
let X : Int_ Sort = 21 in
   let Answer : Int_ Sort = X+X in
      let X : Int_ Sort = 666 in
         WITMOLTUAE (Answer)
```

the parameter value for process WITMOLTUAE is 42

3.2.2 Local Processes

- processes may refer to other processes which are purely local to them, defined using '**where**'; for example:

```
process LIFE
   [birth, puberty, death, marriage, children] : exit :=

   BIOLOGY [birth, puberty, death]
|[puberty, death]|
   FAMILY [puberty, marriage, children, death]

   where

   process BIOLOGY [birth, puberty, death] : exit :=

      birth;
      (
         (puberty; stop)
      [>
         (death; exit)
```

```
    )

  endproc



  process FAMILY
    [puberty, marriage, children, death] : exit :=

    (
      (
        puberty;
        (
          (marriage; exit)
        []
          exit
        )
      )
    |[puberty]|
      (
        puberty;
        (
          (children; exit)
        []
          exit
        )
      )
    )
  |[marriage]|
    (
      (marriage; stop)
    [>
      (death; exit)
    )

  endproc

endproc
```

note that this allows **children** before **marriage** and after **death**; it also excludes infant death in behaviours which do not deadlock!

- the local processes may reference each other and the enclosing process; the enclosing process may reference the local processes

3.2.3 Domains

- events (strictly, gates) and variables may be allowed to range over a set of values; for example:

```
Day in [Mon, Tues, Wed, Thurs, Fri, Sat, Sun]

Prince : Con_Sort
```

- such domains may be used to distribute the following operators over a set:

  [] ||| || |[...]|

- LOTOS restricts the use of these operators with domains; for example:

  choice push, pull : Nat_Sort []
  AMPLIFIER (push, pull)

  choice chan **in** [send, receive], gate **in** [garden, lych] []
  BUFFER [chan, gate]

  par input **in** [line1, line2, line3, line4] |[control]|
  MULTIPLEXER [input, control]

- only one kind of domain (gates or values) may be used at a time

- note that the specified operator indicates what is being distributed, so that the first example above should *not* be parsed as:

  (**choice** push, pull : Nat_Sort)
  []
  (AMPLIFIER (push, pull))

- the use of a domain is equivalent to writing out the behaviour expression which follows for each of the event gates or values in the set; for example:

  choice event **in** [write, read] []
  choice B : Bit_Sort []
  event; MEMORY (B)

  is equivalent to:

  write; MEMORY (0)
  []
  write; MEMORY (1)
  []
  read; MEMORY (0)
  []
  read; MEMORY (1)

3.3 Guards

- behaviour may be made conditional by using the *guard operator* '[...] ->'; for example:

```
[Bonus > 5000] -> ACCEPT_JOB
```

- if the conditional expression in the guard evaluates to **true** then the following behaviour is permitted; if the conditional expression evaluates to **false** then the whole behaviour expression is equivalent to **stop**
- this operator may be used to achieve the effect of '**if...then...else**' or '**case**', such as might be found in a programming language; for example:

```
  ( [Temperature > 0] -> THAW )
[]
  ( [Temperature <= 0] -> FREEZE )
```

```
  ( [Weather willbe Sunny] -> WEAR_T_SHIRT )
[]
  ( [Weather willbe Rainy] -> TAKE_UMBRELLA )
[]
  ( [Weather willbe Snowy] -> WEAR_ANORAK )
[]
  ( [( (Weather wontbe Sunny) and (Weather wontbe Rainy) ) and
      (Weather wontbe Snowy)] -> WEAR_JACKET )
```

- a guard may also be an equation between values; for example:

```
[core_meltdown = imminent] -> SCRAM
```

- note that arithmetic operations (e.g. '<=') and boolean operations (e.g. 'and') are not built into LOTOS; normally, they would be defined in a library, but it is explained later in these course notes how operations can be defined

3.4 Events with Values

3.4.1 Action Denotation

- an *action denotation* is the event part of an action prefix; so far in these notes, only events which name an event gate have been considered
- in LOTOS, an explicit event properly specifies an event gate and a finite sequence of values; for example:

```
tick

dial ! emergency

statistics ! 36 ! 24 ! 36
```

- it is also convenient to specify sets of values; this is called an *extended action denotation*; for example:

  coord ? X : Int_Sort ? Y : Int_Sort

  colour ? Prim : Primary_Sort

  but this is only a short-hand for writing out a choice of event offers with '!' for each value; the last example above is thus equivalent to a choice of:

  colour ! Red

  colour ! Yellow

  colour ! Blue

- both '!' and '?' forms may be mixed; for example:

  calendar ? Month : Month_Sort ! 1986

  note that a multi-part event like this is still atomic: its values are not established in a sequence of events; it is therefore not equivalent to:

  calendar ? Month : Month_Sort; calendar ! 1986

- an event may also be followed by a *selection predicate* – a constraint which restricts the values offered; for example:

  p ? x, y, z : Nat_Sort [SQR (x) + SQR (y) = SQR (z)]

  will accept only those triples of values which satisfy the Pythagorean equation; the shorthand notation used here for '?' is equivalent to:

  p ? x : Nat_Sort ? y : Nat_Sort ? z : Nat_Sort

- only the gate name in events is given in constructions such as:

  |[gate1, gate2]|

  hide g2 **in** PROC [g1, g2]

  choice gate **in** [puerta, Tor]

  par gate **in** [way, crash]

3.4.2 Synchronisation Rules

- only event offers which are identical may synchronise; for example:

  take ! 5

  may synchronise with:

  take ! (2 + 3)

  assuming normal rules for numbers, but not with:

  give ! 5

  take ! 8

  take ! 2 ! 3

  take ! 5 ! 0

  take ! five

- because the '?' form of event is equivalent to a choice between a set of values, '?' may synchronise with '!' if the set of values includes the specifically offered value; for example:

  meeting ? Day : Working_Day_Sort

  may synchronise with:

  meeting ! Tuesday

  but not with:

  meeting ! Sunday

- a further possibility is that both event offers are of the '?' form; in this case, the effect is of an agreement on a value from the intersection of the sets of values (if this is non-empty), provided the values have the same sort

  for example:

  holiday ? Month : Month_Sort [Is_Summer (Month)]

may synchronise with:

```
holiday ? Mon : Month_Sort [Has_31_Days (Mon)]
```

to offer a choice of:

```
holiday ! July
holiday ! August
```

whereas:

```
weight ? W : Int_Sort [W > 0]
```

will not synchronise with:

```
weight ? W : Int_Sort [W < 0]
```

3.5 Processes with Values

3.5.1 Parameters and Results

- a process may be given *formal parameters* as a list of variables and their sorts; their scope is the behaviour definition of the process
  for example:

```
process ADDER [input, output] (Base : Nat0_Sort) : noexit :=

   input ? Offset : Nat0_Sort;
   output ! Base + Offset;
   ADDER [input, output] (Base)

endproc
```

- a process may also yield a list of results; these are written as a list of sort names *only* following **exit** in the header of the process definition
- the results of a process are given using **exit** in the body of the process definition; for example:

```
process EMPTY (list : List_Sort) : exit (Bool_Sort) :=

   [eq (list, nil)] -> exit (true)
[]
   [neq (list, nil)] -> exit (false)

endproc
```

- the results of a process are made available with the '>>' operator using '**accept** ... **in**' to match the results list of the process; for example:

```
process SUM [ip] (Num : Nat0_Sort) : exit (Int_Sort) :=

  (
    [Num = 0] -> exit (0)
  )
[]
  (
    [Num > 0] ->
      ip ? Val : Int_Sort;
      (
        SUM [ip] (Num - 1)
      >>
        accept Total : Int_Sort in
          exit (Val + Total)
      )
  )

endproc
```

3.5.2 Functionality

- the *functionality* of a process is the set of tuples of results it can produce; for example:

```
process DIVIDE [source] (Divisor : Nat_Sort) : exit (Int_Sort, Nat0_Sort) :=
```

has functionality:

$$Int_Sort \times Nat0_Sort$$

- generally speaking, if the behaviour expressions combined by some operator can terminate, their functionality must be the same; for example, **P1** and **P2** must have the same functionality in the following cases:

```
P1 [] P2

P1 [> P2

P1 ||| P2

P1 || P2

P1 |[...]| P2
```

similarly, the functionality of a process must be the same as for each:

```
exit (...)
```

used in its definition, or for a corresponding:

```
>> accept ... in
```

- if a process may terminate with no results, just **exit** is written in its definition header
- if a process cannot terminate, **noexit** is written in its definition header
- in some circumstances it is convenient to specify that any value of a sort may be a result; for example:

```
exit (blonde, any Height_Sort, nubile)
```

- this could be used to specify the independent production of parts of a result; for example:

```
  (...; exit (age, any Sex_Sort) )
|||
  (...; exit (any Nat0_Sort, sex) )
```

where **age** and **sex** are determined independently by some behaviour expressions in parallel; **any** is needed because parallel processes which terminate must exit with the same result

3.6 Summary

- values are defined and used in LOTOS as follows:

variable	Brighton : Re_Sort
expression	(Tom + Jerry) - Tweetie_Pie
local variable	**let** Tel : No_Sort = 73171 **in**
value domain	**choice** Treaty : Nat0_Sort [] PEACE (Treaty)
guard	[Sentry] -> BOX (Buck_Palace)

- the following related concepts are used:

local process	**where process** TICK [tock]
gate domain	**choice** channel **in** [English, Bristol] [] SWIM [channel]
	par interface **in** [V24, V35] ||| CONNECT [interface]

- events are specified as an event gate followed by specific values ('!') or sets of values ('?'):

```
gate ! 13 ? Open : Bool_Sort
```

event offers may synchronise if they are identical; '?' is equivalent to a choice between '!' with specific values

- processes may have parameters and results:

```
process COMPARE [input] (Ref : Int_Sort) : exit (Bool_Sort, Bool_Sort) :=

   input ? Val : Int_Sort;
   exit (Val >= Ref, Val <= Ref)

endproc
```

process results are made available with:

```
   COMPARE [data_in] (-40)
>>
   accept Not_Less, Not_More : Bool_Sort in
      BRANCH [data_out] (Not_Less, Not_More)
```

- the functionality of a process is determined by the sorts of its results; the functionality of terminating behaviour expressions must match in the following cases:

```
[]

[>

|||

||

|[...]|

exit (...)

>> accept ... in
```

3.7 Exercises

- give a simpler behaviour expression equivalent to:

```
(
   average ? Val : Nat0_Sort [Val < 3]; stop
)
||
(
   choice medium in [ether, average, spiritualist] []
      choice Num : Nat0_Sort []
         medium ! Num [Num > 1]; stop
)
```

- what is the permissible behaviour of:

  (gait ! 3; **exit** (Int)) || CONFUSING [gait] (1)

  and also of:

  (gait ! 1; **exit** (Int)) || CONFUSING [gate] (2)

  where **Int** is some variable defined to be of type **Int_ Sort**, and process CONFUSING is defined by:

```
process CONFUSING [gate] (Var : Int_ Sort ) : exit (Int_ Sort) :=

  let Var : Int_ Sort = Var - 2 in
    [Var > 0] ->
      gate ? Var : Int_ Sort;
      exit (Var + 2)
  []
    [Var < 0] ->
      gate ? Var : Int_ Sort;
      exit (2 - Var)

endproc
```

- in the following example, **Next** and **Break** are constants in some sort representing signal values; after considering a specific example such as:

```
  FIB [t, r] (1, 1)
||
  (
    t ! Next; r ? n : Nat_ Sort;
    t ! Next; r ? n : Nat_ Sort; stop
  )
```

  explain in general what the following process may do:

```
process FIB [trigger, result] (Seed1, Seed2 : Nat_ Sort) : noexit:=

  (
    (
      trigger ! Next;
      result ! Seed1;
      exit (Seed2, Seed1 + Seed2)
    )
  []
    (
      trigger ? Seed1, Seed2 : Nat_ Sort;
```

```
            result ! Break;
            exit (Seed1, Seed2)
        )
    )
>>
    accept Seed1, Seed2 : Nat_Sort in
        FIB [trigger, result] (Seed1, Seed2)

endproc
```

3.8 Possible Solutions

- the permitted behaviour is equivalent to:

  average ! 2; **stop**

- for the first case, the permissible behaviour is:

  gait ! 3; **exit** (-1)

  provided **Int** is -1; otherwise it is:

  gait ! 3; **stop**

  for the second case, the permissible behaviour is only:

  stop

- process FIB may accept a **Next** request at gate **trigger**, and output the next number in a Fibonacci series at the gate **result**

  FIB is initialised with two seed values, but may input new values at the gate **trigger**; in this case it outputs **Break** to divide the sequences

  FIB may thus output partial Fibonacci series at the **result** gate such as:

  1, 1, 2, 3, 5, Break

  and:

  3, 7, Break,
  Break,
  217894, 65932, 283826, 349758, Break

Chapter 4

Data Typing

4.1 Abstract Data Types

- a *concrete data type* is a representation of structured data within a computer, e.g. a linked list or the layout of a file control block

- an ADT (*Abstract Data Type*) is an implementation-independent representation of structured data; only the essential characteristics of the data type are specified

- for example, the concrete notion of a book may be described in terms of the quality and colour of its binding and paper, the type of printing ink, the font and size of lettering, etc.

- in abstract terms, a book may be thought of in terms of its components, how these are related, and what may be done to it; its components may be:

  Cover, Title_Page, Text, Index

  and these components may be further decomposed; for example:

  Title, Author, Date, Publisher

- for **Title_Page**, the relationships between the components may include:

  a Title_Page has one Title and one Publisher

  a Title_Page may bear several Authors and Dates

  a book is also characterised by what is done to it; for example:

  Print, Read, Plagiarise, Burn

4.2 Type Definitions

- as explained earlier, a sort is a set of data values; a *type definition* packages up a set of sorts, and specifies the operations on them

  for example:

  type Writer **is** ... **endtype**

- do not confuse the name of the *type* with the names of the *sort(s)* it defines
- typically, simpler types are built into more complex ones using **is** to import the definitions in another type; this is called *enrichment*

  for example:

  type Natural_Number **is**

  ...

  endtype

  type Boolean **is**

  ...

  endtype

  type Arithmetic **is** Natural_Number, Boolean

  ...

  endtype

  type Fraction **is** Arithmetic

  ...

  endtype

- a type definition contains a number of sections in the following order, each section being optional:

  sorts which gives the names of the sorts defined in the type

  opns which gives the names and functionality of the operations on the sorts

  eqns which gives the equations that the operations satisfy

4.3 Operations

- the *operations* on a sort are functions, defined by their *signature*; for example:

increment : Nat_ Sort -> Nat_ Sort

member : Element_ Sort, Set_ Sort -> Bool_ Sort

an operation may have several sorts as its domain, but has exactly one sort as its codomain

- an operation must be defined as total, i.e must be defined for all values in its domain
- several operations may have the same signature; for example:

  tomorrow, yesterday : Day_ Sort -> Day_ Sort

- operations which have a null domain are *constants*; for example:

  0 : -> Nat0_ Sort

  scarlet, vermilion, crimson : -> Red_ Sort

- operations may be defined as *prefix* or *binary infix*; for example:

  is_ instructive : Lecture_ Sort -> Bool_ Sort

  _ * _ : Int_ Sort, Int_ Sort -> Int_ Sort

  where the '_' stands for a positional parameter; for example:

  pred (0) * succ (0)

- one operation may be chosen as the basic *constructor* for a sort; there is often a 'null' constant which is the simplest value
  for example:

  add_ to_ dir : Entry_ Sort, Dir_ Sort -> Dir_ Sort

  empty_ dir : -> Dir_ Sort

- operations which extract components of a sort are called *selectors*; for example:

  name : Entry_ Sort -> Alpha_ Sort

  address : Entry_ Sort -> Alphanumeric_ Sort

  telephone : Entry_ Sort -> Numeric_ Sort

4.4 Equations

4.4.1 Basic Equations

- an operation is specified by giving a list of the *equations* which give the constraints governing it; equations are separated or terminated by ';'
- equations may use free variables; for example:

  forall e : Entry_Sort, d : Dir_Sort

- for the previous example of a telephone directory, the operation:

  length_of_dir : Dir_Sort -> Nat0_Sort

  might have equations:

  length_of_dir (empty_dir) = 0;

  length_of_dir (add_to_dir (e, d)) = length_of_dir (d) + 1

- equations may, if they have the right properties, be regarded as *left-to-right rewrite rules*: if an expression matches the pattern on the left side of an equation it may be replaced by the right side, preserving the same pattern matches
- for example, consider the following operations:

  nil_ac : -> Account_Sort
  credit : Nat_Sort, Account_Sort -> Account_Sort
  debit : Nat_Sort, Account_Sort -> Account_Sort
  balance : Account_Sort -> Int_Sort
  _ + _ , _ - _ : Int_Sort, Nat_Sort -> Int_Sort

  and equations:

  balance (nil_ac) = 0;

  balance (credit (n, a)) = (balance (a)) + n;

  balance (debit (n, a)) = (balance (a)) - n

  which would require the usual definition of numbers

- for this example, an expression such as:

```
balance (debit (10, credit (5, nil_ac)))
```

would be re-written by the third equation above as:

```
(balance (credit (5, nil_ac))) - 10
```

then by the second equation as:

```
((balance (nil_ac)) + 5) - 10
```

and finally by the first equation as:

```
((0) + 5) - 10
```

- with definitions for the decimal representation of numbers and arithmetic, this would be further rewritten to the equivalent of -5

4.4.2 Conditional Equations

- an equation may be made *conditional* by prefixing it with a boolean condition, like a guard for a behaviour expression; for example:

```
m >= n => max (m, n) = m;
m < n    => max (m, n) = n
```

- an equation may also be used instead of a boolean condition; for example:

```
e1 = e2 => sub_from_dir (e1, add_to_dir (e2, d)) = d
```

- as with guards, conditional equations can be used like **if...then...else** or **case** statements
- there is a subtle and important distinction between boolean conditions and equational equality; a boolean condition as a guard is equivalent to writing an equation as a guard

  for example:

```
condition => LHS = RHS
```

  is equivalent to:

condition = true => LHS = RHS

the equational equality for such a guard holds if the equation can be derived from the equations given in the type definition

- only *equality* may be derived from the original equations, not *inequality*; thus, although a boolean condition such as:

  not ((3 * 2) < 7) => wrong = right

  may be negated, an equational equality such as:

  (b1 ≠ b2) => b1 exor b2 = true

  may *not* be

- it is therefore usually necessary to define a boolean equality relation (say, '==') between values of a sort; for example, for the natural numbers:

  opns

0	: -> Nat0_Sort
succ	: Nat0_Sort -> Nat0_Sort
_ == _	: Nat0_Sort, Nat0_Sort -> Bool_Sort

  with the following equations:

  forall n, n1, n2 : Nat0_Sort

0 == 0	= true;
succ (n) == 0	= false;
0 == succ (n)	= false;
succ (n1) == succ (n2)	= n1 == n2

4.5 Overloading

- it is permissible for an operation to be *overloaded*, i.e. given more than one definition; this is generally used when the operations are essentially the same but operate on different sorts

  for example:

_ + _ : Nat0_Sort, Nat0_Sort -> Nat0_Sort

_ + _ : Matrix_Sort, Matrix_Sort -> Matrix_Sort

- sometimes, an overloaded operation may have different domain sorts in more than one definition but the same codomain sort; for example:

  size : List_Sort -> Nat0_Sort

  size : Dir_Sort -> Nat0_Sort

- similarly, the domains may be the same but the codomains may differ; in such cases, the context may indicate which version of the operation is intended

  for example:

  forall e : Entry_Sort, d : Dir_Sort

  size (add_to_dir (e, d)) = size (d) + 1

- however, the context may not always be sufficient to distinguish which version is meant; in this case, the sort of a value or the result of an operation must be explicitly stated using '**of**'

  for example:

  5 * (((temperature - 32) \ 9) **of** Int_Sort)

- similarly, for equations it may be logically necessary to use '**ofsort**' to indicate the sort of the outermost operation in an equation; for example:

  ofsort Bit_Sort

  forall b1, b2 : Bit_Sort

  b1 implies b2 = not (b1) or b2

  however, LOTOS requires that the sort of an equation be given explicitly

- **ofsort** governs the equations which follow up to the next **ofsort**

- **forall** likewise governs the equations which follow; except for the initial, global definitions it must follow **ofsort** if it is needed

4.6 Making New Types

4.6.1 Renaming

- a new type may be created simply by renaming an existing one; this may give new names to sorts and operations using '**sortnames**...**for**...' and '**opnnames**...**for**...'

  for example:

```
type Flag is Bool renamedby

  sortnames

    Flag_Sort for Bool_Sort

  opnnames

    set for true
    clear for false
    toggle for not
    equiv for ==

endtype
```

- note the spelling of **opnnames**

4.6.2 Parameterisation

- it is frequently necessary to define data types which have the same basic structure and operations but a variety of data elements; LOTOS permits *parameterisation* of data types to cater for this
- a parameterised data type has *formal parameters*; for example:

```
type Double_blank is

  formalsorts Thing_Sort

  formalopns

    Z : -> Thing_Sort
    next : Thing_Sort -> Thing_Sort
    _ p _ : Thing_Sort, Thing_Sort -> Thing_Sort

  formaleqns

    forall T, T1, T2 : Thing_Sort

      ofsort Thing_Sort

        T p Z = T;
        T1 p (next (T2)) = next (T1 p T2)

  opns
```

```
    double : Thing_Sort -> Thing_Sort

  eqns

    forall T : Thing_Sort

      ofsort Thing_Sort

        double (T) = T p T

endtype
```

4.6.3 Actualisation

- a parameterised type is *actualised* by a specific type to yield a new type
- the formal sorts and operations must be replaced by actual sorts and operations which are compatible with the signatures and equations of the formal ones; for the example given above, a suitable type for actualisation would be:

```
type Addition is Nat0

  opns
    _ + _ : Nat0_Sort, Nat0_Sort -> Nat0_Sort

  eqns

    forall N, N1, N2 : Nat0_Sort

      ofsort Nat0_Sort

        N + 0 = N;
        N1 + succ (N2) = succ (N1 + N2)

endtype
```

- for this example, the actualisation may be carried out by:

```
type Duplication is Double_blank actualizedby Addition using

  sortnames

    Nat0_Sort for Thing_Sort

  opnnames

    0       for Z
    succ    for next
    +       for p

endtype
```

- note the spelling of *actualizedby*

4.7 Summary

- abstract data types are implementation-independent representations of structured data:

  type Font **is** Letters ... **endtype**

- an abstract data type is characterised by:

```
      sorts   sorts It_takes_all_Sorts
 operations   opns
                Fork          : -> Cutlery_Sort
                shiny         : Cutlery_Sort -> Bool_Sort
                _ picks_up _ : Philosopher_Sort, Cutlery_Sort -> Bool_Sort
  equations   eqns
                forall N, N1, N2 : Nat0_Sort
                  ofsort Nat0_Sort
                    N * 0              = 0;
                    N1 * succ (N2) = (N1 * N2) + N1
```

- equations may be made conditional by prefixing them with a boolean condition or an equation:

  in_Estelle (spec) => conciseness (spec) = low

  lang (spec) = LOTOS => writer (spec) = Professor

- operations may be overloaded, which may mean that the sort of a result has to be explicitly stated:

  Coord **of** X_Coord_Sort

  ofsort French_Sort
  translation (sweetie_pie) = toto

- data types may be renamed:

  Livre **renamedby**

  sortnames

  Book_Sort **for** Livre_Sorte

  opnnames

  read **for** lire
  write **for** ecrire

- parameterised data types are defined using formal parameters:

  formalsorts

  formalopns

  formaleqns

- parameterised data types may be instantiated:

  Glue **actualizedby** Paste **using**

  sortnames

  Flour_Sort **for** Binder_Sort

  opnnames

  smell **for** sniff

4.8 Exercises

4.8.1 Improper Fractions

- assume the type **Arithmetic** defines:
  - sort **Nat_Sort**
  - operations '+' (addition of natural numbers), '*' (multiplication of natural numbers), and '==' (boolean equality of natural numbers)
- define the type **Fractions** to have:
  - constructor **pair** (make a fraction from its numerator and denominator)
  - selectors **numer** and **denom** (select the numerator and denominator of a fraction respectively)
  - operations '+' (addition of fractions) and '==' (boolean equality of fractions)

4.8.2 Washing Machine

- a manually-operated machine has operations:
  - **load** (load the clothes)
  - **wash** (make the clothes clean)
  - **spin** (get rid of most of the water from the clothes)
  - **tumble** (get rid of all the water from the clothes)
- specify this machine, and also the predicates:
  - **clean** (clothes washed at least once)
  - **soaking** (clothes washed but not spun or tumble-dried since then)
  - **dry** (clothes tumble-dried but not washed since then)
- it is presumed that only dirty and dry clothes are initially loaded into the machine

4.8.3 Set of Incrementable Bits

- specify the data type **Bit**:
  - sort **Bit_Sort** with values **0** and **1**
  - operations **flip** (binary complement) and '==' (boolean equality)
- specify the data type **Set_with_Incr**, a parameterised set with a special 'increment' operation:
  - formal sort **Obj_Sort** (for the elements to be instantiated)
  - formal operations **incr** (increment) and **same** (boolean equality of objects)
  - sort **Set_Sort**
  - operations **empty_set** (the null set), **add_to_set** (add an element to a set), and **in_set** (check for set membership)
- using these definitions, specify the data type **Bit_Set_with_Incr** by actualising the type **Set_with_Incr** with **Bit**

4.9 Possible Solutions

4.9.1 Improper Fractions

```
type Fractions is Arithmetic

  sorts Frac_Sort

  opns

    pair            : Nat_Sort, Nat_Sort -> Frac_Sort
    numer, denom    : Frac_Sort -> Nat_Sort
    _ + _           : Frac_Sort, Frac_Sort -> Frac_Sort
    _ == _          : Frac_Sort, Frac_Sort -> Bool_Sort

  eqns

    forall n, d : Nat_Sort, f1, f2 : Frac_Sort

      ofsort Nat_Sort

        numer (pair (n, d))     = n;
        denom (pair (n, d))     = d

    ofsort Frac_Sort

      f1 + f2 =
        pair ((numer (f1) * denom (f2)) + (numer (f2) * denom (f1)),
              denom (f1) * denom (f2))

    ofsort Bool_Sort

      f1 == f2 =
        (numer (f1) * denom (f2)) == (numer (f2) * denom (f1))

endtype
```

4.9.2 Washing Machine

```
type Washing_Machine is Bool

  sorts

    WM_Sort

  opns

    load                    : -> WM_Sort
    wash, spin, tumble      : WM_Sort -> WM_Sort
    clean, soaking, dry     : WM_Sort -> Bool_Sort

  eqns

    forall wm : WM_Sort

      ofsort Bool_Sort

        clean (load)            = false;
        clean (wash (wm))       = true;
        clean (spin (wm))       = clean (wm);
        clean (tumble (wm))     = clean (wm);

        soaking (load)          = false;
        soaking (wash (wm))     = true;
        soaking (spin (wm))     = false;
        soaking (tumble (wm))   = false;

        dry (load)              = true;
        dry (wash (wm)          = false;
        dry (spin (wm))         = dry (wm);
        dry (tumble (wm))       = true

endtype
```

4.9.3 Set of Incrementable Bits

```
type Bit is Bool

  sorts
    Bit_Sort

  opns
    0, 1        : -> Bit_Sort
    flip        : Bit_Sort -> Bit_Sort
    _ == _      : Bit_Sort, Bit_Sort -> Bool_Sort

  eqns

    ofsort Bit_Sort
```

```
        flip (0)    = 1;
        flip (1)    = 0

      ofsort Bool_Sort

        0 == 0      = true;
        0 == 1      = false;
        1 == 0      = false;
        1 == 1      = true

endtype



type Set_with_Incr is Bool

  formalsorts Obj_Sort

  formalopns
    incr : Obj_Sort -> Obj_Sort
    _ same _ : Obj_Sort, Obj_Sort -> Bool_Sort

  formaleqns
    forall o, o1, o2, o3 : Obj_Sort

      ofsort Bool_Sort
        o same o = true;
        o1 same o2 = o2 same o1;
        o1 same o2, o2 same o3 =>
          o1 same o3 = true

  sorts Set_Sort

  opns
    empty_set : -> Set_Sort
    add_to_set : Obj_Sort, Set_Sort -> Set_Sort
    in_set : Obj_Sort, Set_Sort -> Bool_Sort

  eqns
    forall o, o1, o2 : Obj_Sort, s : Set_Sort

      ofsort Bool_Sort
        in_set (o, empty_set) = false;
        o1 same o2 =>
          in_set (o1, add_to_set (o2, s)) = true ;
        not (o1 same o2) =>
          in_set (o1, add_to_set (o2, s)) = in_set (o1, s)

      ofsort Set_Sort
        in_set (o, s) =>
          add_to_set (o, s) = s

      (* other axioms are needed to ensure that the order of
         addition of elements to the set is irrelevant *)

endtype
```

```
type Bit_Set_with_Incr is Set_with_Incr actualizedby Bit using
  sortnames
    Bit_Sort     for Obj_Sort
  opnnames
    flip         for incr
    ==           for same
endtype
```

Chapter 5

Putting It Together

5.1 Complete Specifications

- a complete LOTOS specification has a form such as:

```
specification IN_SPEC [data] (Limit : Time_Sort) : exit

  behaviour CONTROL [data] (Limit) where

  type Character is
    ...
  endtype (* Character *)

  type Word is Character
    ...
  endtype (* Word *)

  process CONTROL [chan] (Real : Time_Sort) : exit :=

    ...

    where

    process STATION [Crewe, Euston] (T : Train_Set) : exit :=

      ...

      where

        process PLATFORM [P1, P2, P3] : exit :=
          ...
        endproc (* PLATFORM *)

    endproc (* STATION *)

    process CONTROLLER [fat, thin] : exit (Engine_Sort) :=

      ...

    endproc (* CONTROLLER *)
```

endproc (* CONTROL *)

endspec (* IN_ SPEC *)

- a specification has formal gates parameters and results just like a process; the keyword **behaviour** (also spelled **behavior**) gives the behaviour expression which characterises the whole specification
- global type definitions are introduced before behaviour definitions; other type and process definitions come in any order
- type definitions may also be introduced as local, following **where**
- type definitions in the standard LOTOS library may be included implicitly; for example:

library

Set, HexDigit

endlib

- informal *comments* may be introduced; for example:

(* This piece of text may be safely ignored *)

- the specification must, of course, be meaningful with all comments deleted; it is a good idea to separate the comments clearly from the formal text

5.2 Semantic Basis

5.2.1 Process Semantics

- the semantics of behaviour expressions is given by *action predicates*; for example:

$$B - gv_1 ::: v_n - > B'$$

which postulates that behaviour expression B may participate in event $g!v_1 ::: !v_n$ and then behave like B'

- for successful termination there is a similar predicate:

$$B - ¡v_1 ::: v_n - > B'$$

and for internal action:

$$B - \mathbf{i} - > B'$$

- the semantics of LOTOS operators are given using *derivation rules*, which have a set of premisses and a conclusion; for example:

$$\frac{G; B - e - > B'}{([G] - > B) - e - > B'}$$

this says that if boolean guard G is *true*, and behaviour expression B may engage in event **e** and then behave like B', then so may $([G] - > B)$

- several derivation rules may be required to give the semantics of an operator; for example:
  - if **e** is not ¡ (i.e. the **exit** event):

$$\frac{B1 - e- > B1'}{B1 [> B2 - e- > B1' [> B2}$$

  - if **e** is ¡ (so that B' must be **stop**):

$$\frac{B1 - ¡- > B1'}{B1 [> B2 - ¡- > B1'}$$

  - in all cases:

$$\frac{B2 - e- > B2'}{B1 [> B2 - e- > B2'}$$

- given the definitions of the operators and the definition of equivalence or congruence, various *algebraic laws* may be deduced which govern the effect of the operators; for example:

$$B \,|||\, \mathbf{exit} = B$$

$$B1 \,||\, B2 = B2 \,||\, B1$$

$$B \,[]\, \mathbf{stop} = B$$

$$(B1 \,[]\, B2) \,[]\, B3 = B1 \,[]\, (B2 \,[]\, B3)$$

$$(e;\, B1) \,[>\, B2 = (e;\, (B1 \,[>\, B2)) \,[]\, B2$$

5.2.2 Data Type Semantics

- the sorts and the operations on them generate an *algebra* of terms; for example:

```
Empty_Pot      mix (Red, mix (Yellow, Empty_Pot))
```

- the semantics of an ACT ONE data type is given by the *initial algebra*, which makes only those terms equal which are required by the equations to be equal
- for example, a data type may require that:

```
(c1 IsIn p) =>
   mix (c1, mix (c2, p)) = mix (c2, p);

mix (c1, mix (c2, p)) = mix (c2, mix (c1, p))
```

the following terms would therefore be made equal in the initial algebra:

mix (Red, mix (Yellow, mix (Red, Empty_Pot)))

mix (Red, mix (Yellow, Empty_Pot))

- initial algebras have a simple operational interpretation using the equations as *left-to-right rewrite rules*; a 'good' set of equations always re-writes expressions to the yield a *canonical* ('standard') form in a finite sequence of steps
- executability of a data type definition requires that the right-hand sides of equations are 'simpler' than the left-hand sides; for example:

  N1 + succ (N2) = succ (N1 + N2)

  is a suitable equation because the right-hand side can be further simplified to yield an ultimate expression of the form:

  succ (... (succ (0)))

  where there are $N1 + N2 + 1$ such **succ** operations

5.3 Summary

- process semantics are given by:

  action predicates

$$B - gv_1 ::: v_n - > B'$$

$$B - ¡v_1 ::: v_n - > B'$$

$$B - \mathbf{i} - > B'$$

  derivation rules

$$\frac{B1 \; - e - > B1'}{B1 \, [] B2 - e - > B1'}$$

- data type semantics are given by the *initial algebra* which makes only those terms equal which are required to be equal by the equations
- LOTOS is a *formal specification language* with features for describing *static* (data type) and *dynamic* (process) aspects of behaviour
- LOTOS is a *general-purpose language* for specifying *concurrent systems*, but was designed to be suitable for specifying OSI services and protocols
- all the examples so far have been of familiar, whimsical or mathematical systems; the following example illustrates how LOTOS can be applied to the specification of communications systems

5.4 A Large Example

5.4.1 Network Service

- the following specification covers data transfer in the *OSI Connection-Mode Network Service*; although large by comparison with other examples in these notes, only a part of the full specification is included
- the architecture of the OSI Network Service is shown in Figure 5.1

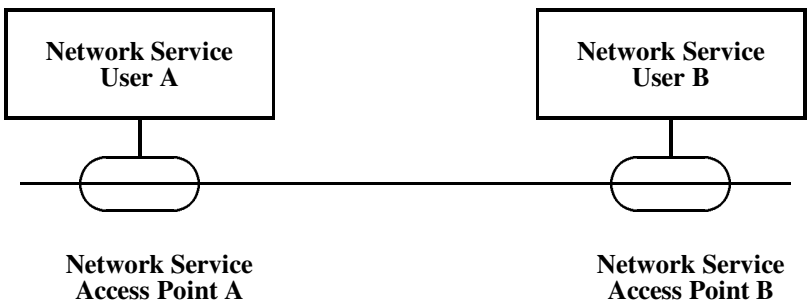


Figure 5.1: Network Service Access Points

- the *Network Service Provider* offers the Network Service to a set of *Network Service Users* through a set of NSAPs (*Network Service Access Points*) which can be thought of as abstract interfaces; NSAPs are identified by a *Network Address* which is unique within the whole Network Service
- in the *Connection-Mode* Network Service, each Network Service User is attached to one NSAP, as shown in Figure 5.2; multiple connections are allowed through an NSAP

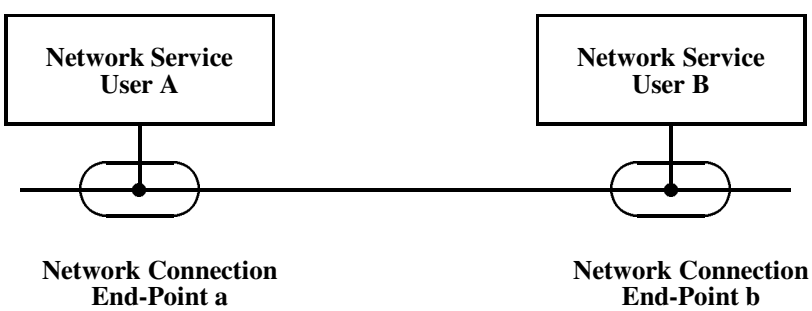


Figure 5.2: Network Connection End-Points

- each connection terminates in an NCEP (*Network Connection End-Point*); NCEPs are distinguished by a *Network Connection End-Point Identifier* which is unique within an NSAP
- a Network Service User interacts with a Network Service Provider by means of *Network Service Primitives* (which can be thought of as abstract interface interactions); two kinds of primitives are identified in this example, as shown in Figure 5.3

5.4.2 Data Transfer

- for this example, data transfer should be specified for just one connection between NSAP **A**/NCEP **a** and NSAP **B**/NCEP **b**

Figure 5.3: Network Service Primitive Types

- once a connection has been established, data is transferred by means of the following service primitives:

  N-DATA request (NS_ User_ Data)

  N-DATA indication (NS_ User_ Data)

  NS_ User_ Data is transferred transparently from one User to the other; one or more *octets* (bytes) of data may be sent

- such *normal data* is subject to *flow-control*, whereby the Service Provider may not accept further data if the connection pipe-line is currently full (the connection capacity may vary and cannot be determined)
- special service primitives are used to send *expedited data*, which will bypass a flow control blockage for normal data:

  N-EXPEDITED-DATA request (NS_ User_ Data)

  N-EXPEDITED-DATA indication (NS_ User_ Data)

  which allow 1 to 32 octets of data to be sent transparently

- normal data is delivered in the same order as it is transmitted; expedited data is similarly delivered in the same order
- however, expedited data may overtake normal data which is in transit, but it is not guaranteed to do so; the only guarantee given is that normal data which is submitted *after* expedited data will not be delivered *before* it

5.4.3 Suggested Data Type Model

- the following library sorts may be assumed:

Boolean	Booleans (sort **Bool**; constants **true** and **false**; operations **and**, **or**, and **not**)
NaturalNumber	natural numbers (sort **Nat**; operations **ge** (greater than or equal to), and **le** (less than or equal to))
OctetString	strings of octets (sort **OctetString**; operations + (concatenation of an octet and an octet string) and **Length** (number of octets))
DecNatRepr	decimal representation of natural numbers (sort **DecimalDigit**; constants **0**, ... **9**; operations **Dec** (decimal digit to string element) and **NatNum** (decimal digit string to natural number)

- the following additional types should be defined:

Addr	Network Addresses, with values **SAP_ A**, and **SAP_ B**
Ident	Network Connection End-Point Identifiers, with values **CEP_ a**, and **CEP_ b**

Data based on **OctetString** and **DecNatRepr**, defining Network Service Data Units as octet strings, and defining **1** and **32** as constants with the usual decimal interpretations

- **NSP_ Sort** should be defined for Network Service Primitives, with the following operations as constructors:

  NDT_ req, **NDT_ ind**

  NEX_ req, **NEX_ ind**

- Network Service Primitives correspond to Network Service Objects, which are the pieces of information which are transferred by the underlying medium; a *request* creates an object, which is later turned into an *indication* on delivery
- **NSO_ Sort** should be defined using **NSP_ Sort** as a basis for Network Service Objects, with operations:

  req to turn an **NSP request** into the corresponding **NSO**
  ind to turn an **NSO** into the corresponding **NSP indication**

- *recognisers* should also be defined which check the kind of a primitive or object; for example:

  Is_ NDT_ req (for an NSP)

  Is_ NDT (for an NSO)

- Network Connection data transfer behaviour should first be defined using an ordinary queue ('medium'); a more complex queue which allows expedited data to overtake normal data should then be defined using the operations:

  delete to delete the first instance of a given object in a queue
  Is_ First to check if an object is the first of its type in a queue
  Is_ Reordered to check if a queue is a valid re-ordering of another

5.4.4 Suggested Process Model

- the LOTOS specification should model the Network Connection using processes:

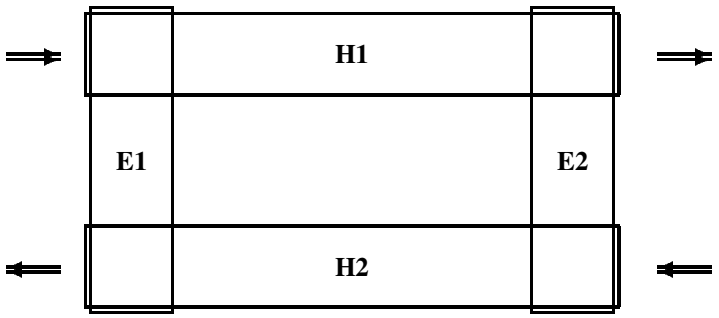
  NCEP which models the behaviour of *one* connection end-point
  NCH which models the behaviour of *one* half-connection (i.e. direction of transfer)

- the first of these processes should be parameterised by the NSAP address and the NCEP identifier; the second process should be parameterised by the NSAP address and NCEP identifier of both ends
- all communication across the Network Service boundary should be defined to take place at one LOTOS gate called **n**; invoking a service primitive should correspond to an event of the form:

  n ! na ! ni ! nsp

  where **na** is an NSAP address, **ni** is an NCEP identifier, and **nsp** is a value constructed for a particular service primitive

- the network connection should be specified by giving the constraints on the behaviour of each end-point and each direction of transfer, as shown in Figure 5.4



Key:

H1, **H2** half-connection constraints

E1, **E2** end-point constraints

Figure 5.4: Network Service Constraints

5.4.5 Possible Solution

```
specification Subset_Network_Service [n] : noexit

  library

    Boolean, NaturalNumber, OctetString, DecNatRepr

  endlib

  type Addr is

    sorts
      Addr_Sort

    opns
      SAP_A, SAP_B : -> Addr_Sort

  endtype (* Addr *)

  type Ident is

    sorts
      Ident_Sort

    opns
      CEP_a, CEP_b : -> Ident_Sort

  endtype (* Ident *)

  type Data is OctetString, DecNatRepr

    opns
      1, 32 : -> Nat

    eqns
```

```
      ofsort Nat
        1 = NatNum (Dec (1));
        32 = NatNum (3 + Dec (2))

endtype (* Data *)


type NSP is Boolean, Data

  sorts
    NSP_ Sort

  opns
    NDT_ req, NDT_ ind,
    NEX_ req, NEX_ ind : OctetString -> NSP_ Sort

    Is_ NDT_ req, Is_ NDT_ ind,
    Is_ NEX_ req, Is_ NEX_ ind : NSP_ Sort -> Bool

    _ eq_ : NSP_ Sort, NSP_ Sort -> Bool

  eqns
    forall dt, dt1, dt2 : OctetString, nsp : NSP_ Sort

      ofsort Bool
        Is_ NDT_ req (NDT_ req (dt)) = true;
        Is_ NDT_ req (NDT_ ind (dt)) = false;
        Is_ NDT_ req (NEX_ req (dt)) = false;
        Is_ NDT_ req (NEX_ ind (dt)) = false;

        Is_ NDT_ req (nsp) =>
          Is_ NDT_ ind (nsp) = false;
        Is_ NDT_ ind (NDT_ ind (dt)) = true;
        Is_ NDT_ ind (NEX_ req (dt)) = false;
        Is_ NDT_ ind (NEX_ ind (dt)) = false;



        Is_ NDT_ req (nsp) or Is_ NDT_ ind (nsp) =>
          Is_ NEX_ req (nsp) = false;
        Is_ NEX_ req (NEX_ req (dt)) = true;
        Is_ NEX_ req (NEX_ ind (dt)) = false;

        Is_ NDT_ req (nsp) or Is_ NDT_ ind (nsp) or Is_ NEX_ req (nsp) =>
          Is_ NEX_ ind (nsp) = false;
        Is_ NEX_ ind (NEX_ ind (dt)) = true;

        not (Is_ NDT_ req (nsp)) =>
          NDT_ req (dt) eq nsp = false;
        NDT_ req (dt1) eq NDT_ req (dt2) = dt1 eq dt2;

        not (Is_ NDT_ ind (nsp)) =>
          NDT_ ind (dt) eq nsp = false;
        NDT_ ind (dt1) eq NDT_ ind (dt2) = dt1 eq dt2;
```

```
            not (Is_NEX_req (nsp)) =>
               NEX_req (dt) eq nsp = false;
            NEX_req (dt1) eq NEX_req (dt2) = dt1 eq dt2;

            not (Is_NEX_ind (nsp)) =>
               NEX_ind (dt) eq nsp = false;
            NEX_ind (dt1) eq NEX_ind (dt2) = dt1 eq dt2

endtype (* NSP *)



type NSO is NSP

   sorts
      NSO_Sort

   opns
      req : NSP_Sort -> NSO_Sort
      ind : NSO_Sort -> NSP_Sort

      Is_NDT, Is_NEX : NSO_Sort -> Bool

      _eq_ : NSO_Sort, NSO_Sort -> Bool

   eqns
      forall dt : OctetString, nsp : NSP_Sort, nso1, nso2 : NSO_Sort

         ofsort NSP_Sort
            ind (req (NDT_req (dt))) = NDT_ind (dt);
            ind (req (NDT_ind (dt))) = NDT_ind (dt);
            ind (req (NEX_req (dt))) = NEX_ind (dt);
            ind (req (NEX_ind (dt))) = NEX_ind (dt)

         ofsort Bool
            Is_NDT (req (nsp)) =
               Is_NDT_req (nsp) or Is_NDT_ind (nsp);
            Is_NEX (req (nsp)) =
               Is_NEX_req (nsp) or Is_NEX_ind (nsp);

            nso1 eq nso2 = ind (nso1) eq ind (nso2)

endtype (* NSO *)



type Basic_Medium is NSO

   sorts
     Medium_Sort

   opns
      empty : -> Medium_Sort
      _app_ : NSO_Sort, Medium_Sort -> Medium_Sort
      _pre_ : Medium_Sort, NSO_Sort -> Medium_Sort
```

```
    _ eq _ : Medium_Sort, Medium_Sort -> Bool

  eqns
    forall nso, nso1, nso2 : NSO_Sort, ncm, ncm1, ncm2 : Medium_Sort

      ofsort Medium_Sort
        nso app empty = empty pre nso;
        (nso1 app (ncm pre nso2)) =
              ((nso1 app ncm) pre nso2)

      ofsort Bool
        empty eq empty = true;
        (ncm pre nso) eq empty = false;
        empty eq (ncm pre nso) = false;
        (ncm1 pre nso1) eq (ncm2 pre nso2) =
          (ncm1 eq ncm2) and (nso1 eq nso2)

endtype (* Basic_Medium *)


type Reordering_Medium is Basic_Medium

  opns
    delete : NSO_Sort, Medium_Sort -> Medium_Sort
    Is_First : NSO_Sort, Medium_Sort -> Bool
    Is_Reordered : Medium_Sort, Medium_Sort -> Bool

  eqns
    forall
      nso, nso1, nso2, : NSO_Sort, ncm, ncm1, ncm2 : Medium_Sort

      ofsort Medium_Sort
        delete (nso, empty) = empty;
        nso1 eq nso2 =>
          delete (nso1, ncm2 pre nso2) = ncm2;
        not (nso1 eq nso2) =>
          delete (nso1, ncm2 pre nso2) =
            delete (nso1, ncm2) pre nso2

      ofsort Bool
        Is_First (nso, empty) = false;
        Is_First (nso1, ncm2 pre nso2) =
          (nso1 eq nso2)
        or
          (
            (
             (Is_NEX (nso1) and Is_NDT (nso2))
            or
             (Is_NDT (nso1) and Is_NEX (nso2))
            )
          and
            Is_First (nso1, ncm2)
          );
```

```
            Is_Reordered (empty, empty) = true;
            Is_Reordered (empty, ncm pre nso) = false;
            Is_Reordered (ncm pre nso, empty) = false;
            Is_Reordered (ncm1 pre nso1, ncm2 pre nso2) =
              (
                (nso1 eq nso2)
              and
                Is_Reordered (ncm1, ncm2)
              )
            or
              (
                (
                 Is_NEX (nso1) and Is_NDT (nso2)
                )
              and
                (
                 Is_First (nso1, ncm2)
                and
                 Is_Reordered (ncm1, delete (nso1, ncm2) pre nso2)
                )
              )

endtype (* Reordering_Medium *)



behaviour
  (
    NCEP [n] (SAP_A, CEP_a)
  |||
    NCEP [n] (SAP_B, CEP_b)
  )
|[n]|
  (
    NCH [n] (SAP_A, SAP_B, CEP_a, CEP_b, empty)
  |||
    NCH [n] (SAP_B, SAP_A, CEP_b, CEP_a, empty)
  )

where



process NCEP
  [n] (na : Addr_Sort, ni : Ident_Sort) : noexit :=

  (
    choice dt : OctetString []
      (
        n ! na ! ni ! NDT_req (dt) [Length (dt) ge 1];
        exit
      []
        n ! na ! ni ! NDT_ind (dt);
        exit
      )
    []
```

```
        (
          n ! na ! ni ! NEX_req (dt)
            [(Length (dt) ge 1) and (Length (dt) le 32)];
          exit
        []
          n ! na ! ni ! NEX_ind (dt);
          exit
        )
    )
  >>
    NCEP [n] (na, ni)

endproc (* NCEP *)



process NCH
    [n] (naX, naY : Addr_Sort, niX, niY : Ident_Sort, ncm1 : Medium_Sort) :
    noexit :=

    (
      (
        choice dt : OctetString, ncm2 : Medium_Sort []
          (
            n ! naX ! niX ! NDT_req (dt);
            exit (req (NDT_req (dt)) app ncm1)
          []
            [ncm1 eq (ncm2 pre req (NDT_req (dt)))] ->
              n ! naY ! niY ! NDT_ind (dt);
              exit (ncm2)
          )
      )
    []
      i;
      (
        choice dt : OctetString, ncm2 : Medium_Sort []
          (
            n ! naX ! niX ! NEX_req (dt);
            exit (req (NEX_req (dt)) app ncm1)
          []
            [ncm1 eq (ncm2 pre req (NEX_req (dt)))] ->
              n ! naY ! niY ! NEX_ind (dt);
              exit (ncm2)
          []
            exit (ncm1)
          )
      )
    )
  >>



    accept ncm2 : Medium_Sort in
      (
        choice ncm3 : Medium_Sort []
          [Is_Reordered (ncm3, ncm2)] ->
```

```
                i;
                NCH [n] (naX, naY, niX, niY, ncm3)
        )

    endproc (* NCH *)

endspec (* Subset_Network_Service *)
```

Appendix A

Further Reading

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Compound nouns in the index are generally entered under the main noun. Entries for acronyms are indexed under the acronym, but are cross-referenced from the expansion of the acronym. Non-alphabetic entries in the index are sorted according to their ASCII code. The page number of a main entry or definition is given in **bold**, while the page number of an example is given in *italics*. Most main entries also give an example.

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